# Homework 4 Solutions

## Section 8.1

#11) Find the exact length of the curve $y = \frac{x^3}{3} + \frac{1}{4x}$ on $1 \le x \le 2$.

$$y' = x^2 - \frac{1}{4x^2}$$

$$(y')^2 = x^4 - \frac{1}{2} + \frac{1}{16x^4}$$

$$L = \int_1^2 \sqrt{1 + x^4 + \frac{1}{16x^4} - \frac{1}{2}}\, dx$$

$$= \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\, dx = \int_1^2 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\, dx$$

$$= \int_1^2 \left(x^2 + \frac{1}{4}x^{-2}\right) dx$$

$$= \left(\frac{x^3}{3} - \frac{1}{4x}\Big|_1^2\right) = \frac{8}{3} - \frac{1}{8} - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= \frac{7}{3} + \frac{1}{8} = \frac{56+3}{24} = \frac{59}{24}$$

#14) Find the exact length of $y = \ln(\cos x)$ on $0 \le x \le \pi/3$.

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

$$(y')^2 = \tan^2 x$$

$$L = \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} \sec x\, dx = \ln|\tan x + \sec x|\Big|_0^{\pi/3}$$

$$= \ln|\sqrt{3} + 2| + \ln(1) = \ln(2+\sqrt{3})$$

## Section 8.2

#9) Find the exact area of the surface obtained by rotating $y^2 = x+1$ on $0 \le x \le 3$ about the x-axis.

$$y^2 - 1 = x$$

$$2y = \frac{dx}{dy}$$

$$S = \int_1^2 2\pi y \sqrt{1+4y^2}\, dy = \frac{\pi}{4}\int_1^2 u^{1/2}\, du$$

$x = 0$: $y = \pm 1$

$x = 3$: $y = \pm 2$

$$u = 1 + 4y^2$$

$$du = 8y\, dy$$

$$= \frac{\pi}{6}\left((1+4y^2)^{3/2}\Big|_1^2\right)$$

$$= \frac{\pi}{6}\left(17^{3/2} - 5^{3/2}\right)$$

#18) Find the area of the surface obtained by rotating $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ on $1 \le x \le 2$ about the y-axis.

$y' = \frac{1}{2}x - \frac{1}{2x}$,

$(y')^2 = \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}$

$$S = \int_1^2 2\pi x \sqrt{1 + \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \int_1^2 \frac{1}{2}x^2 + \frac{1}{2}\, dx$$

$$= \pi\left(\frac{x^3}{3} + x \Big|_1^2\right) = \pi\left(\frac{8}{3} + 2 - \left(\frac{1}{3} + 1\right)\right)$$

$$= \pi\left(\frac{7}{3} + 1\right) = \frac{10\pi}{3}$$

## Section 8.3

#24) Find the moments $M_x$ and $M_y$ and the center of mass of system given $m_1 = 5$ $m_2 = 4$ $m_3 = 3$ $m_4 = 6$; $P_1(-4,2)$ $P_2(0,5)$ $P_3(3,2)$ $P_4(1,-2)$.

$$M_x = \sum_{i=1}^{4} m_i y_i$$
$$= 5(2) + 4(5) + 3(2) + 6(-2)$$
$$= 10 + 20 + 6 - 12 = 24$$

$$M_y = \sum_{i=1}^{4} m_i x_i$$
$$= 5(-4) + 4(0) + 3(3) + 6(1)$$
$$= -20 + 9 + 6 = -5$$

$m = 5 + 4 + 3 + 6 = 18$

Center of Mass is $\left(\frac{4}{3}, -\frac{5}{18}\right)$.

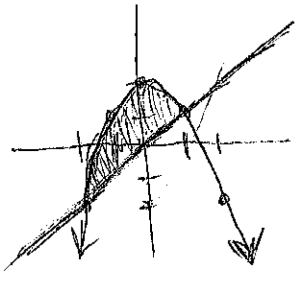
#30) Find the centroid of the region bounded by $y = 2 - x^2$ and $y = x$.

$2 - x^2 = x$

$0 = x^2 + x - 2$

$= (x-1)(x+2)$

$\Rightarrow$ curves intersect at $x = 1$ and $x = -2$



$$A = \int_{-2}^{1} 2 - x^2 - x\, dx = \left(2x - \frac{x^3}{3} - \frac{x^2}{2} \Big|_{-2}^{1}\right)$$
$$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right)$$
$$= 8 - 3 - \frac{1}{2} = \frac{9}{2}$$

The Centriod of the region is $\left(-\frac{1}{2}, \frac{2}{5}\right)$

$$\bar{x} = \frac{2}{9}\int_{-2}^{1} x(2 - x^2 - x)\, dx$$
$$= \frac{2}{9}\left(x^2 - \frac{x^4}{4} - \frac{x^3}{3} \Big|_{-2}^{1}\right)$$
$$= \frac{2}{9}\left(1 - \frac{1}{4} - \frac{1}{3} - \left(4 - 4 + \frac{8}{3}\right)\right)$$
$$= \frac{2}{9}\left(\frac{3}{4} - 3\right)$$
$$= \frac{2}{9}\left(-\frac{9}{4}\right) = -\frac{1}{2}$$

$$\bar{y} = \frac{1}{9}\int_{-2}^{1} (2 - x^2)^2 - x^2\, dx$$
$$= \frac{1}{9}\int_{-2}^{1} 4 - 5x^2 + x^4\, dx$$
$$= \frac{1}{9}\left(4x - \frac{5}{3}x^3 + \frac{x^5}{5} \Big|_{-2}^{1}\right)$$
$$= \frac{1}{9}\left(4 - \frac{5}{3} + \frac{1}{5} - \left(-8 + \frac{40}{3} - \frac{32}{5}\right)\right)$$

$\frac{1}{9}\left(-3 + \frac{33}{5}\right) = \frac{2}{5}$