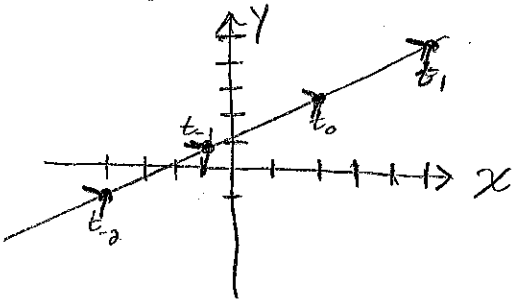
Homework 7 Solutions

Section 10.1

#6) a) Sketch the curve $x = 3t + 2$, $y = 2t + 3$.

t	-3	-2	-1	0	1	2	3
x	-7	-4	-1	2	5	8	11
y	-3	-1	1	3	5	7	9



b) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = 3t + 2 \Rightarrow t = \frac{x-2}{3}$$

$$\therefore y = 2\left(\frac{x-2}{3}\right) + 3 = \frac{2}{3}x - \frac{4}{3} + 3 = \frac{2}{3}x + \frac{5}{3}$$

#15) a) Eliminate the parameter to find a Cartesian equation of the curve.

$x = t^2$

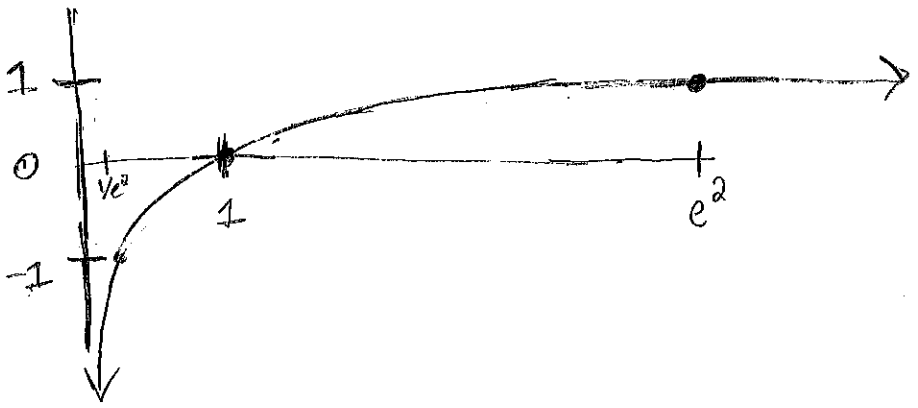
$y = \ln t$

$$x = t^2 \Rightarrow t = \sqrt{x}$$

$$\therefore y = \ln\sqrt{x} = \frac{1}{2}\ln x$$

b) Sketch the curve.

t	$1/e$	1	e
x	$\frac{1}{e^2}$	1	e^2
y	-1	0	1



Section 10.2

#5) Find the equation of the tangent to $x = t\cos t$, $y = t\sin t$ at the Cartesian point corresponding to $t = \pi$.

$dx/dt = \cos t - t\sin t$, $dy/dt = \sin t + t\cos t$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t + t\cos t}{\cos t - t\sin t} \Rightarrow \left.\frac{dy}{dx}\right|_{t=\pi} = \frac{-\pi}{-1} = \pi$$

∴ The slope at $(-\pi, 0)$ is π and the tangent line is $y = \pi(x + \pi)$.

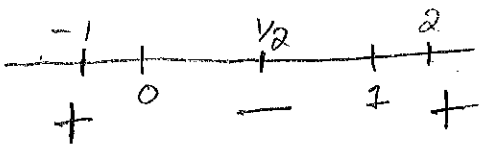
#14) Find dy/dx and d^2y/dx^2 for $x = t^2 + 1$, $y = e^t - 1$. For which values of t is the curve concave upward?

$dx/dt = 2t$, $dy/dt = e^t \Rightarrow \frac{dy}{dx} = \frac{e^t}{2t}$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2te^t - 2e^t}{4t^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{2e^t(t-1)}{4t^3} \cdot \frac{1}{2t} = \frac{e^t(t-1)}{4t^3}$$

The curve is concave up if $\frac{d^2y}{dx^2} > 0$...

When is $\frac{e^t(t-1)}{4t^3} > 0$?



∴ The curve is concave up when $t \in (-\infty, 0) \cup (1, \infty)$.

Section 10.3

#17) Identify the curve by finding a Cartesian equation for $r = 5\cos\theta$.

$x = r\cos\theta \Rightarrow \cos\theta = x/r$. So $r^2 = x^2 + y^2 = 25\cos\theta = 25 \cdot x/5 = 5x$

$$\Rightarrow x^2 - 5x + y^2 = 0 \Rightarrow x^2 - 5x + \frac{25}{4} + y^2 = \frac{25}{4}$$

or $\boxed{(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}}$

#26) Find a polar equation for $x^2 - y^2 = 4$.

$x = r\cos\theta$ and $y = r\sin\theta \Rightarrow (r\cos\theta)^2 - (r\sin\theta)^2 = 4$

$r^2\cos^2\theta - r^2\sin^2\theta = 4$

$r^2(\cos^2\theta - \sin^2\theta) = 4$

$r^2\cos 2\theta = 4$

$r^2 = 4\sec 2\theta$

$r = 2\sqrt{\sec 2\theta}$