Homework #8 Solutions

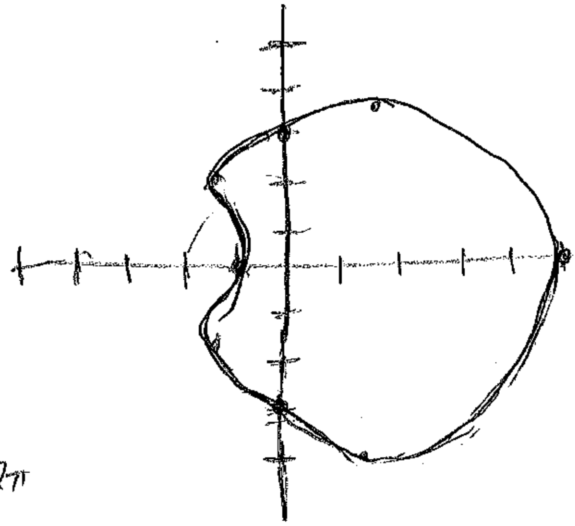
Section 10.4

#11) Sketch the curve $r = 3+2\cos\theta$ and find the enclosed area.

r	0	$\pi/6$	$\pi/3$	$\pi/2$
θ	5	$3+\sqrt{3}$	4	3

r	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$
θ	2	$3-\sqrt{3}$	1	$3-\sqrt{3}$

r	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
θ	2	3	4	$3+\sqrt{3}$



$$A = \frac{1}{2}\int_0^{2\pi}(3+2\cos\theta)\,d\theta = \frac{1}{2}\int_0^{2\pi}(9+12\cos\theta+4\cos^2\theta)\,d\theta$$

$$= \frac{1}{2}\left[9\theta+12\sin\theta\right]\Big|_0^{2\pi} + \int_0^{2\pi}(1+\cos 2\theta)\,d\theta$$

$$= 9\pi + \left(\theta+\frac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi} = \boxed{11\pi}$$

#27) Find the area of the region that lies inside $r = 3\cos\theta$ and outside $r = 1+\cos\theta$.

$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} 9\cos^2\theta\,d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(8\cos^2\theta - 2\cos\theta - 1)\,d\theta = 2\int_{-\pi/3}^{\pi/3}(1+\cos 2\theta)\,d\theta - \left(\sin\theta+\frac{\theta}{2}\right)\Big|_{-\pi/3}^{\pi/3}$$

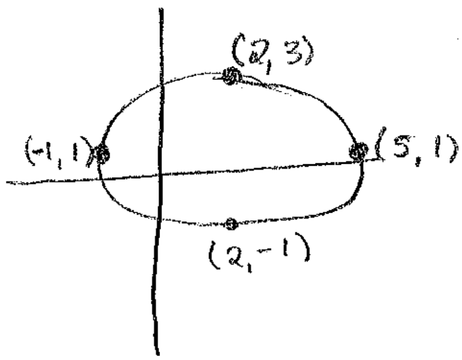
$$= 2\left(\theta+\frac{1}{2}\sin(2\theta)\right)\Big|_{-\pi/3}^{\pi/3} - \left(\sin\theta+\frac{\theta}{2}\right)\Big|_{-\pi/3}^{\pi/3}$$

$$= \frac{2\pi}{3}+\frac{\sqrt{3}}{2}+\frac{2\pi}{3}+\frac{\sqrt{3}}{2} - \left(\frac{\sqrt{3}}{2}+\frac{\pi}{6}+\frac{\sqrt{3}}{2}+\frac{\pi}{6}\right)$$

$$= \frac{4\pi}{3} - \frac{\pi}{3} = \pi$$

Section 10.5

#18) Find an equation of the ellipse and its foci.



Major axis: 3
Minor axis: 2
Center: (2, 1)

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$$

$c^2 = 9 - 4 = 5$
$\Rightarrow c = \pm\sqrt{5}$

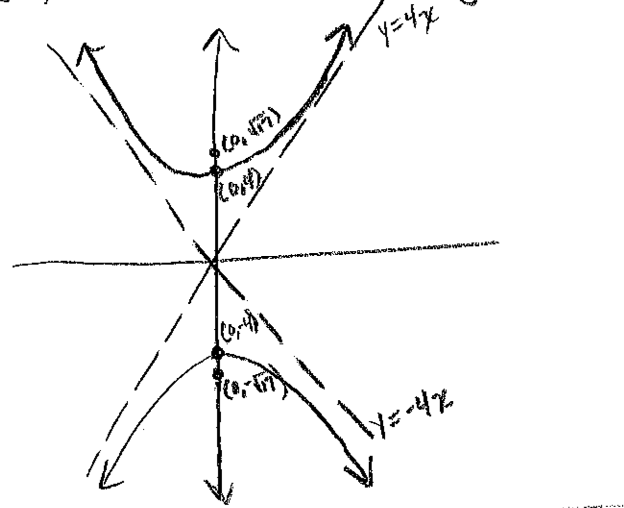
$\therefore$ foci @ $(2 \pm \sqrt{5}, 1)$

#22) Find the vertices, foci, and asymptotes of $y^2 - 16x^2 = 16$. Sketch its graph.

$y^2 - 16x^2 = 16 \Rightarrow \frac{y^2}{4^2} - x^2 = 1 \Rightarrow a = 4 \quad b = 1$

$\Rightarrow c^2 = a^2 + b^2 = 4^2 + 1^2 = 17 \quad \therefore$ foci @ $(0, \pm\sqrt{17})$

The vertices are located at $(0, \pm 4)$ and the asymptotes are at $y = 4x$ and $y = -4x$.



#36) Find the equation of the parabola w/ vertical axis going through the points (0, 4), (1, 3), (-2, -6).

Vertex axis $\Rightarrow$ the standard form of the parabola is $y = ax^2 + bx + c$.

Using the given points, we construst the system $\begin{cases} c = 4 \\ a + b + c = 3 \\ 4a - 2b + c = -6 \end{cases}$ which we can reduce to $\begin{cases} a + b = -1 \\ 4a - 2b = -10 \end{cases}$

$$\begin{array}{l} 2a + \cancel{2b} = -2 \\ 4a - \cancel{2b} = -10 \\ \hline 6a \quad = -12 \\ a = -2 \end{array}$$

$\Rightarrow -2 + b = -1$
$b = 1$

Therefore, the parabola is given by $y = -2x^2 + x + 4$.

#13) Find the eccentricity, identify the conic, given an equation for the directrix, and sketch the conic.

$$r = \frac{9}{6+2\cos\theta} = \frac{3/2}{1+\frac{1}{3}\cos\theta}$$

The eccentricity is $e = \frac{1}{3}$. Therefore, this conic is an ellipse.

$ed = 3/2 \Rightarrow \frac{1}{3}d = 3/2$ or $d = \frac{9}{2}$. Therefore, the directrix is the line $x = \frac{9}{2}$.

$\theta = 0 \Rightarrow r = \frac{9}{8}$, $\theta = \pi \Rightarrow r = \frac{9}{4}$

$\theta = \frac{\pi}{2} \Rightarrow r = 3/2$ $\theta = \frac{3\pi}{2} \Rightarrow r = 3/2$