Homework # 9 Solutions

Section 11.1

#14) Find a formula for a_n of the sequence $\{4, -1, \frac{1}{4}, -\frac{1}{16}, \ldots\}$.

$a_n = (-1)^{n+1}\left(\frac{1}{4}\right)^{n-2}$

$a_1 = (-1)^2\left(\frac{1}{4}\right)^{-1} = 4$

$a_2 = (-1)^3\left(\frac{1}{4}\right)^0 = -1$

$a_4 = (-1)^5\left(\frac{1}{4}\right)^2 = -\frac{1}{16}$

#30) Determine whether $a_n = \dfrac{4^n}{1+9^n}$ converges or diverges. If it converges, find the limit.

$\circledast = \lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{4^n}{1+9^n} = \dfrac{\infty}{\infty}$ so L'H $\Rightarrow$ $\circledast = \lim_{n\to\infty} \dfrac{4^n \cdot \ln 4}{9^n \cdot \ln 9}$

$= \lim_{n\to\infty} \dfrac{\ln 4}{\ln 9} \cdot \left(\dfrac{4}{9}\right)^n = 0$

$\therefore$ a_n converges to 0.

#42) $a_n = \ln(n+1) - \ln n$
$= \ln\left(\frac{n+1}{n}\right)$

Since $\ln x$ is continuous,

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right)$

$= \ln\left(\lim_{n\to\infty} \frac{n+1}{n}\right) = \ln 1 = 0$

Section 11.2

Determine whether the series is convergent or divergent

#24) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n} = \sum_{n=0}^{\infty} \frac{3 \cdot 3^n}{(-2)^n} = \sum_{n=0}^{\infty} 3\left(\frac{-3}{2}\right)^n \to \infty$ since $\left|-\frac{3}{2}\right| > 1$.

#30) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k^2-2k+5}$ $\quad \displaystyle\lim_{k\to\infty} \frac{k^2}{k^2-2k+5} = 1 \neq 0 \Rightarrow$ series diverges by Basic divergence test.

#33) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4+e^{-n}} \cdot \frac{e^n}{e^n} = \sum_{n=1}^{\infty} \frac{e^n}{4e^n+1}$ $\quad \displaystyle\lim_{n\to\infty} \frac{e^n}{4e^n+1} \overset{*}{=} \frac{\infty}{\infty}$

$\overset{L'H}{\Rightarrow} * = \displaystyle\lim_{n\to\infty} \frac{e^n}{4e^n} = \frac{1}{4} \neq 0$ $\therefore$ series diverges by Basic divergence test.