Solutions to Selected Problems — Homework #10

Section 4.9

#36) Given $f'(x) = \frac{x+1}{\sqrt{x}}$ with $f(1)=5$, find f.

$$f'(x) = \frac{x+1}{x^{1/2}} = \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}} = x^{1/2} + x^{-1/2}$$

$$f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

$$5 = f(1) = \frac{2}{3} + 2 + C = \frac{8}{3} + C \Rightarrow C = 5 - \frac{8}{3} = \frac{7}{3}$$

$$\therefore \quad f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + \frac{7}{3}$$

Section 5.1

#21) Use defn 2 to find an expression for the area under the graph of $f(x) = \frac{2x}{x^2+1}$ on $1 \leq x \leq 3$.

$$\Delta x = \frac{3-1}{n} = \frac{2}{n} \Rightarrow A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1+\frac{2}{n}i\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2\left(1+\frac{2}{n}i\right)}{\left(1+\frac{2}{n}i\right)^2+1} \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2+\frac{4i}{n}}{1+\frac{4i}{n}+\frac{4i^2}{n^2}+1} \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\frac{4}{n}+\frac{8i}{n^2}}{\frac{4i^2}{n^2}+\frac{4i}{n}+2} \cdot \frac{n^2}{n^2}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4n+8i}{4i^2+4in+2n^2}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2n+4i}{2i^2+2in+n^2}\right)$$

Section 5.2

#48) If $\int_2^8 f(x)\,dx = 7.3$ and $\int_2^4 f(x)\,dx = 5.9$, find $\int_4^8 f(x)\,dx$.

$$\int_2^8 f(x)\,dx = \int_2^4 f(x)\,dx + \int_4^8 f(x)\,dx$$

$$7.3 = 5.9 + \int_4^8 f(x)\,dx$$

$$\Rightarrow \int_4^8 f(x)\,dx = 7.3 - 5.9 = 1.4$$

#56) Use properties of the integral to verify that $\int_0^1 \sqrt{1+x^2}\,dx \le \int_0^1 \sqrt{1+x}\,dx$.

Note that $0 \le x \le 1. \Rightarrow x^2 < x.$

Then $\sqrt{1+x^2} \le \sqrt{1+x}$ on $[0,1]$.

Thus, $\int_0^1 \sqrt{1+x^2}\,dx \le \int_0^1 \sqrt{1+x}\,dx$

Section 5.3

#18) Use FTC to differentiate $y = \int_{\sin x}^1 \sqrt{1+t^2}\,dt$.

$$y = -\int_1^{\sin x} \sqrt{1+t^2}\,dt \Rightarrow y' = -\cos x\sqrt{1+\sin^2 x}$$

#27) Evaluate $\int_0^1 (u+2)(u-3)\,du$.

$$\int_0^1 (u+2)(u-3)\,du = \int_0^1 (u^2 - u - 6)\,du$$

$$= \left(\frac{1}{3}u^3 - \frac{u^2}{2} - 6u\right)\Big|_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{2} - 6\right) - 0$$

$$= -\frac{1}{6} - 6 = \frac{-37}{6}$$