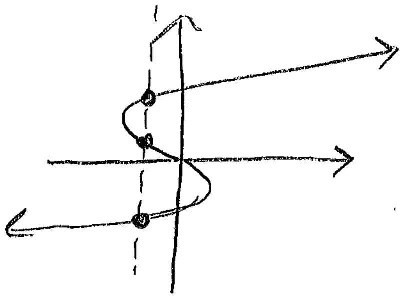
# HMWK 1 KEY

Calc 1

## Section 1.1

7) Determine whether the following curve is the graph of a function of $x$.



There is a vertical line that intersection the curve more than once.
Thus, the curve is not a function of $x$ since it fails the VLT.

27) Evaluate the difference quotient for $f(x) = 4+3x-x^2$ at $x=3$.

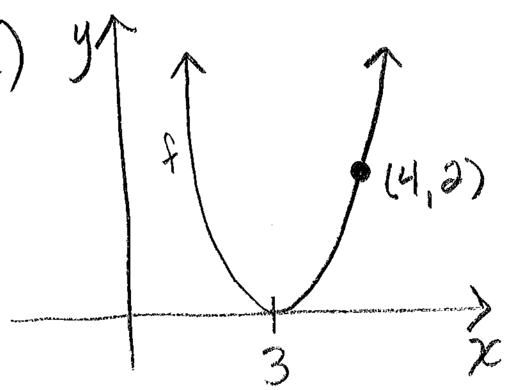
$$f(3+h) = 4+3(3+h)-(3+h)^2 = 4+9+3h-(9+6h+h^2)$$
$$= 4-3h-h^2$$
$$f(3) = 4+3(3)-(3)^2 = 4$$

$$\frac{f(3+h)-f(3)}{h} = \frac{\cancel{4}-3h-h^2\cancel{-4}}{h} = \frac{-h(h+3)}{h} = \boxed{-h-3}$$

## Section 1.2

10) Find the expression of the following quadratic functions.

a)



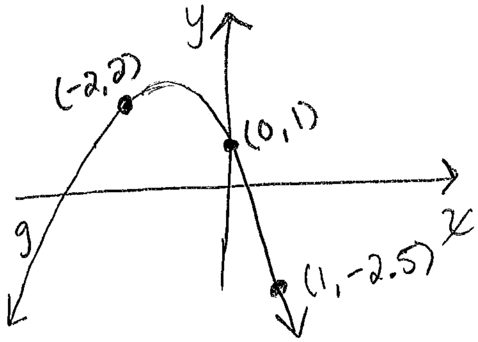

The vertex of this parabola is at $(3,0)$.

Vertex form: $y = a(x-h)^2+k$

$$2 = a(4-3)^2+0 \Rightarrow a=2$$

$$\therefore f(x) = 2(x-3)^2$$

b)




Standard form: $y = ax^2 + bx + c$

$2 = a(-2)^2 + b(-2) + c \Rightarrow 2 = 4a - 2b + c$

$1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$-2.5 = a(1)^2 + b(1) + c \Rightarrow -2.5 = a + b + c$

Since $c = 1$, $1 = 4a - 2b$ and $-3.5 = a + b$

Thus, we must solve a system of two equations in two unknowns.

$$\begin{aligned} 1 &= 4a - 2b \\ -3.5 &= a + b \end{aligned} \Rightarrow \begin{aligned} 1 &= 4a - \cancel{2b} \\ -7 &= 2a + \cancel{2b} \\ \hline -6 &= 6a \end{aligned} \Rightarrow a = -1$$

$\Rightarrow -3.5 = -1 + b$ or $b = -2.5$

$\therefore g(x) = -x^2 - 2.5x + 1$

## Section 1.3

63) ⓐ If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function $f$ such that $f \circ g = h$.

Note that $g$ is invertible. $x = 2y + 1 \Rightarrow g^{-1}(x) = \frac{x-1}{2}$

So $f = f \circ g \circ g^{-1} = h \circ g^{-1} \Rightarrow f(x) = h(g^{-1}(x))$

$\Rightarrow f(x) = 4\left(\frac{x-1}{2}\right)^2 + 4\left(\frac{x-1}{2}\right) + 7$

$= 4 \cdot \frac{(x-1)^2}{4} + 2(x-1) + 7$

$= x^2 - \cancel{2x} + 1 + \cancel{2x} - 2 + 7$

$= x^2 + 6$

(b) If $f(x) = 3x+5$ and $h(x) = 3x^2+3x+2$, find a function $g$ such that $f \circ g = h$.

$f$ is linear and thus invertible... $x = 3y+5 \Rightarrow f^{-1}(x) = \frac{x-5}{3}$

So $g = f^{-1} \circ f \circ g = f^{-1} \circ h$.

$\Rightarrow g(x) = f^{-1}(h(x)) = \frac{3x^2+3x+2-5}{3} = \frac{3(x^2+x-1)}{3} = x^2+x-1$

## Section 1.5

10) Determine whether $f(x) = x^4 - 16$ is one-to-one.

(pf) Suppose $f(x) = f(y)$ for some $x, y \in \mathbb{R}$.

Then $x^4 - 16 = y^4 - 16 \Rightarrow x^4 = y^4$ and $x = \pm y$.

Thus, $f(-y) = f(y)$ showing that $f$ is not one-to-one. ∎

26) Find a formula for the inverse of $y = \frac{1-e^{-x}}{1+e^{-x}}$.

First, we must show $y$ is invertible..

$$\frac{1-e^{-x_1}}{1+e^{-x_1}} = \frac{1-e^{-x_2}}{1+e^{-x_2}} \Rightarrow (1-e^{-x_1})(1+e^{-x_2}) = (1-e^{-x_2})(1+e^{-x_1})$$

$$\Rightarrow \cancel{1} - e^{-x_1} + e^{-x_2} - \cancel{e^{-x_1-x_2}} = \cancel{1} + e^{-x_1} - e^{-x_2} - \cancel{e^{-x_1-x_2}}$$

$$\Rightarrow 2e^{-x_1} = 2e^{-x_2} \Rightarrow e^{-x_1} = e^{-x_2}$$

Taking the natural log and dividing by $-1$ on both sides gives $x_1 = x_2$. So $y$ is indeed invertible.

As standard, we switch $x$ and $y$ and solve for $y$ again.

$x = \frac{1-e^{-y}}{1+e^{-y}} \Rightarrow x(1+e^{-y}) = 1-e^{-y} \Rightarrow e^{-y}(1+x) = 1-x$

$\Rightarrow e^{-y} = \frac{1-x}{1+x}$. Taking natural log of both sides yields

$-y = \ln\left(\frac{1-x}{1+x}\right) \Rightarrow \boxed{y = -\ln\left(\frac{1-x}{1+x}\right)}$