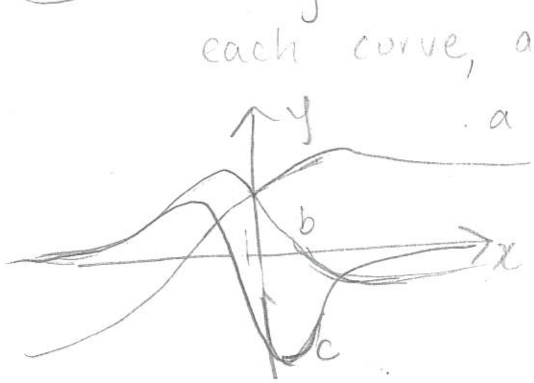
Section 2.8

(49) The figure shows graphs of f, f', and f''. Identify each curve, and explain your choices.



$f \equiv a \quad f' \equiv b \quad f'' \equiv c$.

Note that when a increases when b is positive and decreases when b is negative. Thus, b is the derivative of a. Similarly, b increases/decreases corresponding to when c is positive/negative, respectively.

(63) Recall that a function f is called even if $f(-x) = f(x)$ for all x in its domain and odd if $f(-x) = -f(x)$ for all such x. Prove each of the following.

a) The derivative of an even function is an odd function.

pf) Let f be an even function.

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{f(x-h) - f(x)}{h}$$

$$= -\lim_{h \to 0} \frac{f(x) - f(x-h)}{h} = -f'(x)$$

Hence, the derivative is odd.

b) The derivative of an odd function is an even function.

pf) Let f be an odd function.

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{f(x) - f(x-h)}{h} = f'(x)$$

Hence, the derivative is even.

Section 3.1

(27) Differentiate $G(q) = (1+q^{-1})^2$.

$$G'(q) = 2(1+q^{-1}) \cdot \frac{d}{dq}[q^{-1}] = 2(1+q^{-1}) \cdot (-q^{-2})$$

$$= \frac{-2(1+q^{-1})}{q^2}$$

(57) Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

$y' = 2e^x + 3 + 15x^2$

y has a tangent line with slope 2 iff $2e^x + 3 + 15x^2 = 2$ for some $x \in \mathbb{R}$. That is, $2e^x = -15x^2 - 1$.

Clearly, there is no x-value satisfying this equation since $2e^x > 0$ for all x and $-15x^2 - 1 < 0$ for all x.

Therefore, y has no tangent line with slope 2.

Section 3.2

(27) Find $f'(x)$ and $f''(x)$ given that $f(x) = (x^3+1)e^x$.

$$f'(x) = 3x^2 e^x + e^x(x^3+1) = e^x(x^3+3x^2+1)$$

$$f''(x) = e^x(x^3+3x^2+1) + e^x(3x^2+6x) = e^x(x^3+6x^2+6x+1)$$

(41) If $f(x) = \frac{x^2}{1+x}$, find $f''(1)$.

$$f'(x) = \frac{(1+x)\cdot(2x) - x^2 \cdot 1}{(1+x)^2} = \frac{2x+2x^2-x^2}{(1+x)^2} = \frac{x^2+2x}{(1+x)^2}$$

$$f''(x) = \frac{(1+x)^2(2x+2) - (x^2+2x)(2(1+x))}{(1+x)^4} = \frac{2(1+x)((1+x)^2 - (x^2+2x))}{(1+x)^4}$$

$$= \frac{2}{(1+x)^3} \qquad \therefore f''(1) = \frac{2}{(1+1)^3} = \frac{2}{8} = \frac{1}{4}$$