Selected Homework Solutions — Homework 6

Section 3.2

#53 How many tangent lines to the curve $y = \frac{x}{x+1}$ pass through the point $(1,2)$? At which points do these tangent lines touch the curve?

$$y' = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$2 - f(x_0) = \frac{1}{(x_0+1)^2}(1 - x_0) \Rightarrow 2 - \frac{x_0}{x_0+1} = \frac{1-x_0}{(x_0+1)^2}$$

$$2(x_0+1)^2 - x_0(x_0+1) = 1 - x_0$$

$$2x_0^2 + 4x_0 + 2 - x_0^2 - x_0 = 1 - x_0$$

$$x_0^2 + 4x_0 + 1 = 0 \Rightarrow x_0 = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$= -2 \pm \sqrt{3}$$

There are two tangent lines that pass through $(1,2)$. They are at $(-2+\sqrt{3}, f(-2+\sqrt{3}))$ and $(-2-\sqrt{3}, f(-2-\sqrt{3}))$.

Section 3.3

#18 Prove that $\frac{d}{dx}(\sec x) = \sec(x)\tan(x)$.

Note that $\sec x = \frac{1}{\cos x}$.

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\cos(x)\cdot 0 - 1(-\sin(x))}{\cos^2(x)}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec(x)\tan(x)$$

Section 3.4

#15 Differentiate $f(t) = e^{at}\sin(bt)$.

product rule: $ae^{at}\sin(bt) + be^{at}\cos(bt) = f'(t)$

#27 Differentiate $r(t) = 10^{2\sqrt{t}}$.

$$r'(t) = \ln 10 \cdot 10^{2\sqrt{t}} \cdot t^{-1/2}$$

Section 3.5

#27 Find the tangent line to $x^2 - xy - y^2 = 1$ at $(2,1)$.

$$2x - y - x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2x-y}{x+2y}$$

$$\therefore m = \frac{2(2)-1}{2+2(1)} = \frac{3}{4}$$

$$y - 1 = \frac{3}{4}(x-2)$$

#36 Find y'' by implicit differentiation if $x^2 + xy + y^2 = 3$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x+y}{x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(-2 - \frac{dy}{dx}\right) + (2x+y)\left(1 + 2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$\frac{-3y + 3x\frac{dy}{dx}}{(x+2y)^2} = \frac{-2x - \frac{dy}{dx}\cdot x - y - 2y\frac{dy}{dx} + 2x + y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx}}{(x+2y)^2}$$

$$\frac{3\left(x\frac{dy}{dx}-y\right)}{(x+2y)^2} = \frac{3\left(x\left(-\frac{2x+y}{x+2y}\right)-y\right)}{(x+2y)^2}\cdot\frac{x+2y}{x+2y}$$

$$= \frac{3\left(x(-2x-y)-y(x+2y)\right)}{(x+2y)^3}$$

$$= \frac{3(-2x^2+xy-xy-2y^2)}{(x+2y)^3}$$

$$= \frac{-6(x^2+y^2)}{(x+2y)^3}$$

$$= \frac{3(-2x^2-xy-xy-2y^2)}{(x+2y)^3}$$

$$= \frac{-6(x^2+xy+y^2)}{(x+2y)^3} = \frac{-18}{(x+2y)^3}$$