Section 3.4

#61 If $F(x)=f(g(x))$, where $f(-2)=8$, $f'(-2)=4$, $f'(5)=3$, $g(5)=-2$, and $g'(5)=6$, find $F'(5)$.

$$F'(x)=f'(g(x))\cdot g'(x) \Rightarrow F'(5)=f'(g(5))\cdot g'(5)$$
$$= f'(-2)\cdot g'(5)$$
$$= 4\cdot 6 = 24$$

#76 For what values of r does the function $y=e^{rx}$ satisfy the differential equation $y''-4y'+y=0$?

$y'=re^{rx}$ $\quad y''=r^2e^{rx}$

$$\Rightarrow r^2e^{rx}-4re^{rx}+e^{rx}=0$$
$$\underbrace{e^{rx}}_{\text{never zero}}(r^2-4r+1)=0$$

$$\begin{array}{r} 48 \\ -16 \\ \hline 36 \end{array}$$

$$r^2-4r+1=0$$
$$r=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$$

Section 3.5

#76 Find the equation of both tangent lines to the ellipse $x^2+4y^2=36$ that pass through the point $(12,3)$.

$2x+8y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x}{4y}$ A tangent line will satisfy

$3-y_1=-\frac{x_1}{4y_1}(12-x_1) \Rightarrow 12y_1-4y_1^2=x_1^2-12x_1 \Rightarrow x_1^2+4y_1^2=36=12(x_1+y_1)$

$\therefore y_1=3-x_1$. $x_1^2+4(3-x_1)^2=36 \Rightarrow 5x_1^2-24x_1=0 \Rightarrow x=0$ or $x=\frac{24}{5}$

$x=0 \Rightarrow y=\pm3$ i.e. $(0,\pm3)$

$x=\frac{24}{5} \Rightarrow y=\pm\frac{9}{5}$ i.e. $(\frac{24}{5},\pm\frac{9}{5})$

$\boxed{\text{Thus, the tangent lines are } y-3=0 \text{ and } y+\frac{9}{5}=\frac{2}{3}(x-\frac{24}{5}).}$

Note that $(0,-3)$ and $(24,\frac{9}{5})$ aren't actually tangent and through $(12,3)$

$-\frac{24/5}{4(9/5)}$ $\quad \frac{24}{5}\cdot\frac{5}{36}$

Section 3.6

Differentiate the functions.

#9 $g(x) = \ln(xe^{-2x}) = \ln x + \ln e^{-2x} = \ln x - 2x$

$g'(x) = \frac{1}{xe^{-2x}}(e^{-2x} - 2xe^{-2x}) = \frac{1}{x} - 2$

#12 $h(x) = \ln(x + \sqrt{x^2-1})$

$h'(x) = \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{\cancel{x+\sqrt{x^2-1}}}{(\cancel{x+\sqrt{x^2-1}})\sqrt{x^2-1}}$

$= \frac{1}{\sqrt{x^2-1}}$

Section 3.8

#4 A bacteria culture grows with constant relative growth rate. The bacteria count was 400 after 2 hours and 25,600 after 6 hours.

a) What is the relative growth rate?

$400 = y_0 e^{2k}$
$25600 = y_0 e^{6k}$ $\Rightarrow 64 = e^{(6-2)k} = e^{4k} \Rightarrow k = \frac{3}{4}\ln 4 \approx 1.0397$

$\boxed{103.97\%}$

b) What is the initial size of the culture?

$400 = y_0 e^{\frac{3}{2}\ln 4} = 4^{3/2} y_0 = 8y_0 \Rightarrow \boxed{y_0 = 50}$

c) Find an expression for the number of bacteria after t hours.

$y(t) = 50e^{\frac{3}{4}\ln 4\, t} = 50(64)^{t/4}$

e, d) Find the number of cells and growth rate after 4.5 hrs.

$y(4.5) = 50(64)^{4.5/4} \approx 5382$ $\quad y'(t) = \frac{50\ln 64}{4}(64)^{t/4}$

$y'(4.5) \approx 5596$ bacteria/hr

f) When will the population reach 50,000?

$50000 = 50(64)^{t/4} \Rightarrow 1000 = (64)^{t/4} \Rightarrow \frac{4\ln 1000}{\ln 64} = t \approx 6.64$ hours