Iterative Methods with Dynamic Preconditioning for a Moving Spectral Element Technique Applied to the Journal Bearing Problem

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Abstract

A moving spectral element method is described for solving the dynamically loaded journal bearing problem. In this problem a lubricant occupies the region between an inner cylinder which rotates and moves under a time-dependent load and an outer cylinder which is stationary. The path of the journal is tracked in time in order to determine the stability of the bearing and the minimum oil film thickness of the lubricant. Particular emphasis is given to the choice of efficient preconditioners for both the statically and dynamically loaded problems. The effects of cavitation and variable viscosity on the stability of the bearing are discussed.

Key words: moving spectral element method, dynamically loaded journal bearing problem, preconditioners, cavitation, variable viscosity.

AMS subject classifications: 65N35, 65N22, 76A05.

1 Introduction

The journal bearing is an essential part of all internal combustion engines as a means of transferring the energy from the piston rods to the rotating crankshaft. The main journal bearing consists of a journal which is part of the crankshaft and a bearing which is fixed to the engine casing. In general, the two cylinders are eccentrically positioned with a film of lubricating oil separating the two surfaces. The system is subject to dynamic loading in which the journal is allowed to move under the force the fluid imparts on it and the force applied by the engine. Under these forces the centre of the journal traces out a nontrivial locus in space. A knowledge of the path of the journal is important if lubrication engineers are to predict and understand bearing fatigue. Lubrication engineers are interested in determining the global minimum oil film thickness as the journal moves in time in order to assess the role of viscosity on bearing wear.

The traditional approach to the study of journal bearing lubrication has been via the lubrication approximation introduced by Reynolds [18]. This enables an equation for the pressure within the thin film region of the geometry to be written separately from the kinematical and constitutive equations describing the flow of the lubricant, thereby simplifying greatly the calculation of the reaction forces engendered by the lubricant. Whereas the effectiveness of the lubrication approximation has been supported by experimental evidence in a very wide range of lubrication studies, there are at least two contexts in which the approximation may be open to question. The first is in predicting the fine details of the nonlinear dynamics of the journal bearing. Here the precise pressure boundary conditions exploited in the Reynolds equation can have a profound effect on the dynamics of the journal. The second context is in studying the role of viscoelasticity in journal bearing lubrication. Within the lubrication approximation, normal stresses are viscosity dominated, making it difficult to accommodate elastic effects in the analysis. However, when the relaxation time of a viscoelastic lubricant is sufficiently high, for example due to pressure thickening, enhanced normal stresses are possible when the lubrication approximation is not employed [8].

If the lubrication approximation is not invoked, there is no option but to solve the full set of coupled equations governing the flow of the lubricant, taking proper account of the moving parts of the geometry. Until recently this task has proved too formidable a calculation, but with current computing power combined with efficient and accurate nu-

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merical methods, the calculation may be attempted.

In this paper we concentrate on the development of accurate and efficient numerical techniques for solving journal bearing problems. A comprehensive physical interpretation of the results generated by our numerical algorithm is beyond the scope of this paper and will be reported elsewhere. We consider the long bearing approximation in this paper, i.e. we assume that everything is constant along the axis of the bearing. The extension of this work to the finite bearing which is a fully 3-D problem is in progress.

We consider the flow of a non-Newtonian lubricant under dynamic loading conditions. The locus of the centre of the journal is determined by solving the equation of motion of the journal. This calculation requires a knowledge of the force exerted on the journal by the lubricant. Therefore the motion of the journal is coupled to the solution of the governing equations for the flow of a lubricant. Since the journal moves in time the flow geometry for the lubricant also changes in time and therefore a new discretisation mesh is required at each new time step. A spectral element method is used to solve the fluid equations at each time step. The discretised equations are solved by a nested preconditioned conjugate gradient method. A comprehensive discussion of which are the most efficient preconditioners in the journal bearing context is given. For realistic journal bearings the radii of the journal and bearing are very similar, i.e. the clearance between the two is very small. This has severe consequences for the iterative solution of the algebraic equations defined in such geometries and profoundly affects the choice of preconditioner. The effect of cavitation and variable viscosity on the stability of the journal is discussed.

2 Formulation of the problem

2.1 The geometry

Consider the two-dimensional geometry shown schematically in Fig. 1. The journal of radius R_J rotates with a constant angular velocity ω in a stationary bearing of radius R_B. Both the journal and the bearing are assumed to be of infinite extent in the axial z-direction. The distance between the axes of the journal and the bearing is given by e.

The eccentricity ratio is defined by $\epsilon = e/c$, where $c = R_B - R_J$ is the average gap so that $0 \leq \epsilon \leq 1$. The region between the journal and the bearing is occupied by a lubricant. The journal is free to move under the action of an applied load which may be variable, its own weight and the reaction force exerted on it by the lubricant. This means that the centre of the journal traces out a nontrivial path in space.

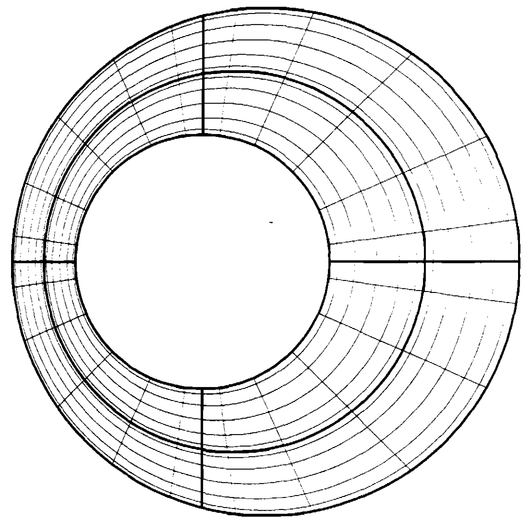

Figure 1: A spectral element discretisation of the journal bearing problem with $E_a = 4$, $E_r = 2$ and $N = 7$.

2.2 The governing equations

The governing equations for a Generalised Newtonian fluid comprise the conservation of momentum

$$(1) \qquad \rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla.\mathbf{T},$$

the conservation of mass

$$(2) \qquad \nabla\cdot\mathbf{v} = 0,$$

and the constitutive equation

$$(3) \qquad \mathbf{T} = 2\eta(\dot\gamma, p)\mathbf{d},$$

where ρ is the density, η is the variable viscosity, $\dot\gamma = \sqrt{2\,\mathrm{tr}(\mathbf{d})^2}$, $\mathbf{T}$ is the extra-stress tensor and $\mathbf{d} = \frac{1}{2}(\nabla\mathbf{v} + (\nabla\mathbf{v})^T)$ is the rate of deformation tensor. Here $\mathrm{tr}(\mathbf{A})$ denotes the trace of a tensor $\mathbf{A}$. The constitutive equation (3) is a modification of the usual Generalised Newtonian model to include pressure dependence of the viscosity.

The viscosity law that we have used was proposed by Li and Davies [8]. It is shear-thinning and pressure-thickening. The various parameters in the model are determined empirically. The dependence of viscosity on $\dot\gamma$ and pressure is given by

$$(4) \quad \eta = \left\{\eta_\infty + \frac{(\eta_0 - \eta_\infty)}{[1 + (K\dot\gamma)^m]}\right\} \times \exp(-\alpha \mathrm{tr}(\boldsymbol{\sigma})/3 + F)$$

where K is a function of pressure

$$K = K(p) = \exp(-\bar{\alpha}\mathrm{tr}(\boldsymbol{\sigma})/3 + E)$$

$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{T}$ is the Cauchy stress tensor and $\eta_0, \eta_\infty, m, n, \alpha, \bar{\alpha}, E$ and F are material parameters which are estimated by best-fitting experimental data. This model describes the shear-thinning behaviour of the viscosity by a Cross-type formula. Pressure-thickening is modelled by a simple exponential law [1]. It is important to note that the viscosity law (4) is consistent with experiments [2] which span only limited ranges of the pressures which the lubricants experience under general operating conditions.

2.3 Cavitation

The assumption that a complete lubricating film is maintained throughout the operation of a journal bearing is well-known to be false in many realistic situations. Under certain conditions the lubricating film ruptures and a cavity is formed. Indeed the very presence of a cavitating region has been shown to be sufficient to stabilise the motion of the journal. Many investigators ([14],[17]) suggest that in the case of a journal operating under the assumption of full-film conditions and without an applied load instability is universal. In this case the journal moves from its equilibrium position, under static loading conditions, in an orbit of growing size until the bearing fails, i.e. the journal and bearing surfaces touch one another. This phenomena is known as whirl instability [13]. In practice the full-film condition is not realistic since in many journal bearing models the large negative pressures produced in the oil cause the oil to vapourise leading to cavitation. There are many ways of modelling cavitation (see Cameron [7], for example), some of which are more sophisticated than others. However, they may all be viewed as ways of circumventing the rather complex two-phase oil-vapour flow problem. Brindley et al. ([3], [4], [5]) have considered a π-film model within the lubrication approximation in the case of a constant applied load. They showed that with the inclusion of a cavitation model there were conditions under which the motion of the journal was stable where previously it was unstable.

In this paper we shall consider two cavitation models. The first is a dual phase variant of the so-called π-film or half-Sommerfeld condition. In the π-film cavitation model it is assumed that the lubricant cavitates in the diverging part of the journal bearing geometry. In its dual phase variant the viscosity of the fluid in the cavitating region is set to be very low while the viscosity of the fluid in the non-cavitating region is kept as that of the lubricant. The second model which we call the single-phase variable-film cavitation model makes no assumption about the size of the cavitating region. The governing equations for the lubricant are solved subject to the full-film assumption and the cavitating region is then determined by the region of subambient pressures. The reaction force on the journal is calculated by integrating the pressure over the non-cavitating region. We shall refer to these models as cavitation models (A) and (B), respectively.

3 Motion of the journal

This is by far the greatest computational challenge in the present study. A single computer run tracking the journal from rest to a periodic equilibrium state takes many hours of DEC-alpha CPU time, under realistic running conditions. Two factors are primarily responsible for the high CPU time. These are the use of small time steps which are required for stability and accuracy of the numerical approximation, typically $O(10^{-8})$ seconds, and the dynamic remeshing due to the translational motion of the journal.

We assume that the centre of mass of the journal behaves as a particle of effective mass M_e situated at the centre of the journal. The equation of motion of the journal is given by

$$M_e\ddot{\mathbf{r}} = \mathbf{F} + \mathbf{R}, \tag{5}$$

where $\mathbf{r}$ is the position vector of the centre of the journal with respect to a coordinate system fixed in space. The applied load, $\mathbf{F}$, in this paper, is taken to be

$$\mathbf{F} = (0, F), \quad F = F_p \sin(\omega t) - M_e g + F_c, \tag{6}$$

where the parameters F_p and F_c allow one to specify the amplitude and mean level of the applied load. The reaction force $\mathbf{R}$ is the force the fluid exerts on the journal and is determined from the solution of the governing equations of motion for the lubricant.

The journal is tracked in time by integrating (5) in the following manner. The flow of the lubricant at each time step is computed using what is known as a quasi-steady approximation. At a given time $t = n\Delta t$, say, the steady flow equations are solved for the current position of the journal using the spectral element method. The force which the fluid exerts on the journal is then calculated by integrating the stresses around the journal,

$$\mathbf{R} = -\int_{\Gamma_J} \boldsymbol{\sigma}.\bar{\mathbf{n}}\, ds,$$

where Γ_J is the surface of the journal and $\bar{\mathbf{n}}$ is the outward unit normal vector to the journal. The right-hand side of

(5) is then updated and the equation integrated in time using the forward Euler method to obtain the new position of the journal at time $t = (n+1)\Delta t$. The process is repeated by solving the steady flow equations in the new region between the journal and the bearing. Note that in this approximation the solutions to the fluid equations at successive time steps are independent of one another. However, in many situations this appears to be a reasonable approximation due to the size of the time step.

4 Moving spectral element algorithm

The region Ω between the journal and bearing is partitioned into a number of spectral elements, Ω_k, $1 \le k \le K$, such that $\cup_{k=1}^{K} \bar{\Omega}_k = \bar{\Omega}$ and $\Omega_k \cap \Omega_l = \emptyset$ for all $k \ne l$, as shown in Fig. 1. Let E_r and E_a denote the number of spectral elements in the radial and azimuthal directions, respectively. We also assume that the decomposition is geometrically conforming in the sense that the intersection of two adjacent elements is either a common vertex or an entire edge. Each physical element is mapped onto the parent element $[-1,1]\times[-1,1]$ on which a Legendre Gauss-Lobatto grid is used. The transfinite mapping technique of Gordon and Hall [10] is used to perform this mapping. The $P_N - P_{N-2}$ spectral element method ([15], [16]) is used in which the velocity and pressure approximations are of degrees N and $N-2$, respectively. The velocity and pressure approximations corresponding to element k are therefore

$$\mathbf{v}_N^k(\xi,\eta) = \sum_{i=0}^{N}\sum_{j=0}^{N} \mathbf{v}_{i,j}^k h_i(\xi) h_j(\eta), \tag{7}$$

$$p_N^k(\xi,\eta) = \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} p_{i,j}^k \tilde{h}_i(\xi) \tilde{h}_j(\eta), \tag{8}$$

respectively. With the velocity and pressure approximation spaces thus chosen the Babuška-Brezzi compatibility condition is satisfied. There are no spurious pressure modes in the pressure approximation.

The discrete variational formulation of (1)-(2) leads to an algebraic system of the form

$$A_\eta \mathbf{v} - D^T \mathbf{p} = \mathbf{g}, \tag{9}$$

$$D\mathbf{v} = \mathbf{h}, \tag{10}$$

where A_η represents the discrete weak form of the diffusion operator $\eta\Delta$ and D is the discrete divergence operator. Block Gaussian elimination yields a symmetric positive definite system for the pressure

$$S\mathbf{p} = \mathbf{c} \tag{11}$$

where $S = DA_\eta^{-1}D^T$ and $\mathbf{c} = -DA_\eta^{-1}\mathbf{g} + \mathbf{h}$. The system (11) is solved for the vector of pressure unknowns using the preconditioned conjugate gradient (PCG) method. Since the system is solved iteratively the stiffness matrices do not need to be set up. Instead they are kept in elemental form. The particular way in which the spectral element grid is constructed ensures that the entries of the pressure matrix S are dependent only on the eccentricity ratio ϵ and not on the orientation of the journal. We denote this dependence by S_ϵ.

The solution of systems of algebraic equations of this form, derived from a spectral element discretisation of the constant viscosity Stokes problem, is well-documented in the literature (see [15], for example) and suitable preconditioners have been advocated which yield efficient numerical algorithms. However, for the solution of realistic journal bearing problems in which $R_B - R_J$ is small and ϵ is near unity the system of equations is extremely ill-conditioned and the spectrum of S_ϵ is not so well-behaved. We shall give an example in the next section, for a constant viscosity lubricant in a statically loaded journal bearing, which shows that this is a result of the extremely large physical aspect ratio for this geometry. The spectral element mesh for geometries of this type comprises elements that are highly distorted. A comprehensive study of preconditioners for this problem is given in [11] in the case of the statically loaded bearing.

For the dynamically loaded bearing the preconditioner is changed dynamically in time taking into account the current eccentricity ratio of the journal and the number of PCG iterations required for convergence at each time step. The preconditioner is based on the pressure matrix evaluated at a given eccentricity ratio. The preconditioner is updated dynamically if a prescribed maximum number of PCG iterations is exceeded [12].

5 Efficient preconditioners

A good preconditioner, P, for S should be an approximation to S, in some sense, which meet the following criteria:

(a) P is easier and cheaper to invert than S;

(b) P is sparse so that it is efficient to construct and store;

(c) the eigenvalues of $P^{-1}S$ are more clustered than those of S.

The last criterion ensures that convergence of the iterative method is rapid. The preconditioner is decomposed in the form $P = QQ^T$ and the transformed system written as

$$Q^{-1}SQ^{-T}(Q^T\mathbf{p}) = Q^{-1}\mathbf{b}. \tag{12}$$

The condition number, κ, of the preconditioned system is defined by

$$\kappa = \frac{\lambda_{max}}{\lambda_{min}} \tag{13}$$

where λ_{max} and λ_{min} are the largest and smallest eigenvalues of the preconditioned matrix L where

$$L = Q^{-1}SQ^{-T}. \tag{14}$$

In practice the steps of the conjugate gradient method are rewritten so that the preconditioner is applied in its entirety.

5.1 Static loading

Initially efficient preconditioners are sought for the statically loaded journal bearing. In this problem the journal rotates about its own axis but is not allowed to move translationally. This means that the geometry is fixed and S remains constant in time. In addition we shall restrict ourselves to a constant viscosity lubricant. In particular the following preconditioners are considered:

(a) $P = B$ where B is the pressure mass matrix.

(b) $P = \hat{B}$ where $\hat{B}$ is the diagonal of B.

(c) $P = S_{\epsilon^*}$ where S_{ϵ^*} is the matrix S_ϵ evaluated at $\epsilon = \epsilon^*$.

The pressure mass matrix B is not diagonal because the quadrature rule used in setting up this discrete variational formulation requires extrapolated values of the pressure on the elemental boundaries. Note that for a spectral element discretisation the pressure mass matrix is not sparse at the elemental level. This choice, given by (a), is widely advocated in the spectral element literature. However, we demonstrate that for the journal bearing problem with realistic values of the parameters this choice does not provide a well-conditioned preconditioned matrix. This means that not only will the resulting algorithm converge slowly but also that the numerical approximation will not be particularly accurate.

The geometry of a car engine journal bearing is such that the typical physical aspect ratio, proportional to $\alpha \equiv c/2\pi R_J$, is very small where $c \equiv R_B - R_J$ is the average gap of the fluid region. Typical values are much less than 1/100. Therefore not even a significant redefinition of the spectral element discretisation would overcome this problem.

We show that the eigenvalue spectrum is very much dependent on the physical aspect ratio. In Figs. 2 and 3 we choose $\epsilon = 0.5$ and keep (N, E_r, E_a) fixed at $(7, 2, 2)$ which gives $M = 196$, where $M{=}E_r{\times}E_a{\times}N^2$ is the size of the pressure matrix S, and show the eigenvalue spectra of the preconditioned system when the preconditioner is the pressure mass matrix i.e. $P = B$. The outer radius in both

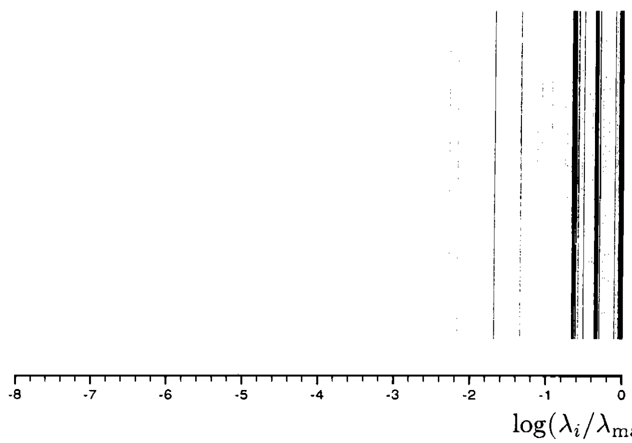

Figure 2: Eigenvalue spectrum of preconditioned system with $P{=}B$ with $(N, E_r, E_a) = (7, 2, 2)$ and illustration of model for the case of $\alpha = 5.3{\times}10^{-2}$, $\epsilon{=}0.5$. The result is a condition number $\kappa{=}188.3$ and number of PCG iterations = 28

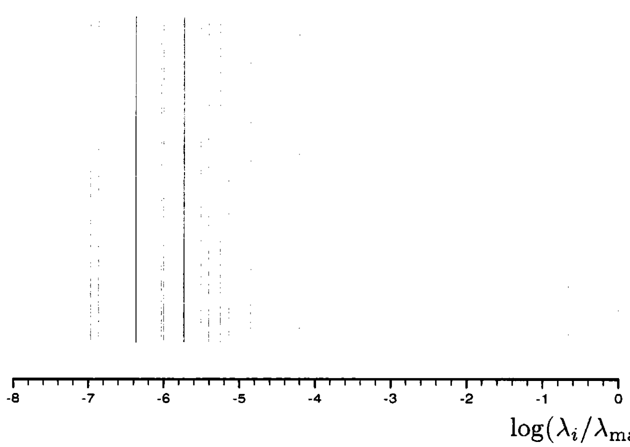

Figure 3: Eigenvalue spectrum of preconditioned system with $P{=}B$ and $(N, E_r, E_a) = (7, 2, 2)$ together with illustration of model for the case of $\alpha = 2.04{\times}10^{-4}$, $\epsilon{=}0.5$. The result is a condition number $\kappa = 9.35 \times 10^6$ and number of PCG iterations = 40

figures is the same (R_B=0.03129) and the inner radius is varied so as to vary α. In Fig. 2, α=0.053 whilst in Fig. 3, $\alpha = 2.04 \times 10^{-4}$. The result is that the condition number increases as α decreases with a cluster of very small eigenvalues appearing in the latter figure. In terms of conjugate gradient iterations the model with α=0.053 takes 28 iterations to converge whilst with $\alpha = 2.04 \times 10^{-4}$, 40 iterations are required. The position is much worse for the preconditioner $P = \hat{B}$ for which the PCG method fails to converge within the theoretical maximum number of M iterations for $\alpha = 2.04 \times 10^{-4}$, indicating that round-off errors dominate. Changing the number of elements or the number of nodes whilst keeping the elemental aspect ratio fixed does not alter the condition number of the preconditioned system with $P = B$. The resulting condition number for both these examples is $\kappa = 9.36 \times 10^6$.

Thus we have shown conclusively that the condition number of the preconditioned system is dependent on the preconditioner and the physical aspect ratio, α, but not on the elemental aspect ratio.

Table 1 compares the condition number and the number of preconditioned conjugate gradient iterations for different preconditioners of S with ϵ=0.5 . It is clear that the most successful preconditioner is $S_{0.0}$ in terms of the number of iterations. This preconditioner needs to be calculated only once and can be applied to systems with different eccentricities.

(N, E_r, E_a), M	P=I	P=$\hat{B}$
(7,2,2), 196	284 (4.30×10^8)	164 (1.79×10^8)
(10,2,2), 400	1162 (8.50×10^8)	395 (3.31×10^8)
(13,2,2), 676	>2000 (1.39×10^9)	727 (5.24×10^8)

(N, E_r, E_a), M	P=B	P=$S_{0.0}$
(7,2,2), 196	40 (9.35×10^6)	29 (55.84)
(10,2,2), 400	63 (9.35×10^6)	34 (55.84)
(13,2,2), 676	95 (9.35×10^6)	37 (55.84)

Table 1: The number of PCG iterations of the preconditioned system with ϵ=0.5 for different preconditioners, P, together with, in brackets, the corresponding condition number. The results are for different numerical parameters based upon the p-method analysis. $\alpha = 2.04 \times 10^{-4}$.

The inversion of the pressure mass matrix also requires a nontrivial amount of work. In addition it needs to be recomputed for different eccentricities. The other two preconditioners (I and $\hat{B}$) although trivial to calculate and invert are very poor preconditioners in terms of the number of PCG iterations required for convergence. In practice, however, we are not limited to using the preconditioner based on the concentric pressure coefficient matrix $S_{0.0}$. It is possible to construct and store a set of preconditioners S_{ϵ_i}, $1 \le i \le I$, for different eccentricities ϵ_i and to use the preconditioner S_{ϵ_j} for which ϵ_j is closest to ϵ. This approach will be explored further for the dynamically loaded journal bearing.

Note that the condition number of the preconditioned system is independent of N for the two cases P=B and P=$S_{0.0}$. In this sense we say that both preconditioners are optimal and that we can expect the number of PCG iterations to be proportional to M. However, as can be seen from the table the number of iterations using both these preconditioners increases less than proportionally with N. This effect is more pronounced in the case P=$S_{0.0}$ where the number of iterations required for N=10 and N=13 is almost the same. Note, however, that the condition number using $P = B$ remains high and therefore numerical results generated using this preconditioner may not be as accurate as those obtained using $P = S_{0.0}$.

5.2 Dynamic loading

The major challenge in this problem in which the journal is free to move translationally in response to an applied load and a force exerted on it by the lubricant is that the geometry is changing in time. The aim is to develop a code which can be used to determine the response of the bearing due to various inputs such as applied load, viscosity of the lubricant, initial position of the journal, speed of rotation of the journal and choice of cavitation model. It is important to determine under which conditions the journal traverses a path that will ultimately bring it into contact with the bearing, thereby causing bearing failure. For our purposes bearing failure occurs when $\epsilon > 0.98$ in which case there is metal-to-metal contact. A typical run takes many thousands of iterations with time steps of $O(10^{-8})$ seconds.

We propose using a preconditioner which changes dynamically in time taking into account the current eccentricity ratio of the journal and the number of PCG iterations required for convergence at each time step. The preconditioner is based on the pressure matrix evaluated at either the present or a previous value of the eccentricity ratio. A given preconditioner may be used for many time steps for which the actual eccentricity ratio will be different to that used to compute the preconditioner. The current preconditioner is changed when the number of iterations required for convergence of the outer conjugate gradient iteration exceeds a prescribed maximum number of iterations. In the results which we present this number was set to be twelve. Once this number has been exceeded a new preconditioner is constructed corresponding to the current eccentricity ratio. The work involved in setting up a new preconditioner is negligible compared with the time taken to track the motion of the journal from its initial state until either the bearing fails or a stable equilibrium point or closed orbit is found. The amount of time spent on the construction of preconditioners is always less than 5% of the total CPU time.

6 Numerical results

In this section we shall present a selection of results showing the influence of cavitation and variable viscosity on the path of the journal. The results which we present are for the choice of discretisation parameters given by $E_r = 1$, $E_a = 4$ and $N = 8$. Variation of the number of elements, the degree of approximation and the time step has been performed to ensure that the final paths are independent of these numerical parameters. The initial position of the

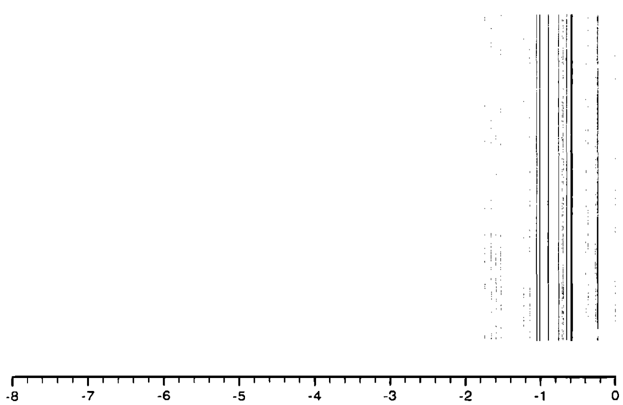

Figure 4: Eigenvalue spectrum of preconditioned system with $P=S_{0.0}$ and $(N, E_r, E_a) = (7,2,2)$, $\alpha = 2.04 \times 10^{-4}$ and ϵ=0.5 . The result is a condition number $\kappa = 55.8$ and number of PCG iterations = 27.

journal is specified by an eccentricity ϵ_o and an attitude angle ϕ_o. In Fig. 5 we compare the paths resulting from the use of cavitation models (A) and (B). In this example we have a variable applied load given by $F_p = 2 \times 10^5$ N/m, $F_c = 0$ N/m, a journal of mass per unit length $M_e = 1.75 \times 10^4$ kg/m and an angular velocity $\omega = 350$ rad/s. In figures of this type the motion of the journal

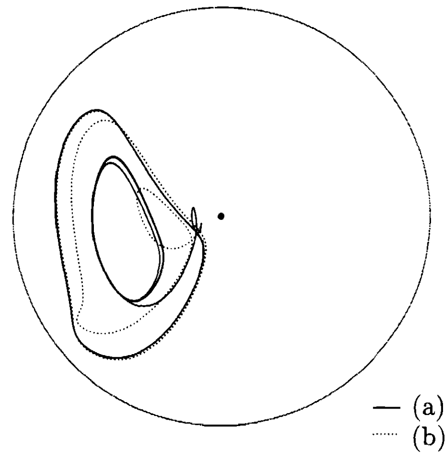

Figure 5: Comparison of the paths of (a) variable-film single-phase and (b) π-film double-phase cavitation models for a journal of $M_e = 1.75 \times 10^4$ kg/m, ω=350rad/s, (ϵ_0,ϕ_0)=(0.10,0.35), $F_p = 2 \times 10^5$ N/m, F_c=0 N/m with time $t \in [0,0.15]$.

is given in eccentricity ratio/attitude angle space. So, for example, if the path travels through the centre of the reference circle then $\epsilon = 0.0$ which means that the journal and bearing are concentric and if the path touches the reference circle then $\epsilon = 1.0$ and so the journal touches the bearing. For each of the models a different closed path is found as the limiting motion of the journal. With reference to Figs. 6 and 7 which are plots of the eccentricity ratio and attitude angle of the journal against time, respectively, we see that the final closed path solution is reached after no more than a handful of orbits of the journal around the bearing. For cavitation model (A) a minimum oil film thickness is attained corresponding to an eccentricity ratio of 0.62 whereas for model (B) it occurs at an eccentricity ratio of 0.90. Even for large magnitude applied loads, such as that used in the present example, the choice of cavitation model is important. Clearly the minimum oil film thickness is sensitive to the choice of cavitation model that is used in the numerical calculations. The rather simplistic π-film model which is used extensively in practice cannot be relied upon to give definitive results although it may well be useful in predicting trends for journal bearing stability. A study of the whirl speed from Fig.7 shows that the periods of both paths are identical to that of the variable applied load.

Finally, we consider the effect of variable viscosity on the stability of the journal bearing. We consider a journal with $M_e = 7 \times 10^4$ kg/m rotating with an angular velocity of 250 rad/s, and subject to an applied load which is

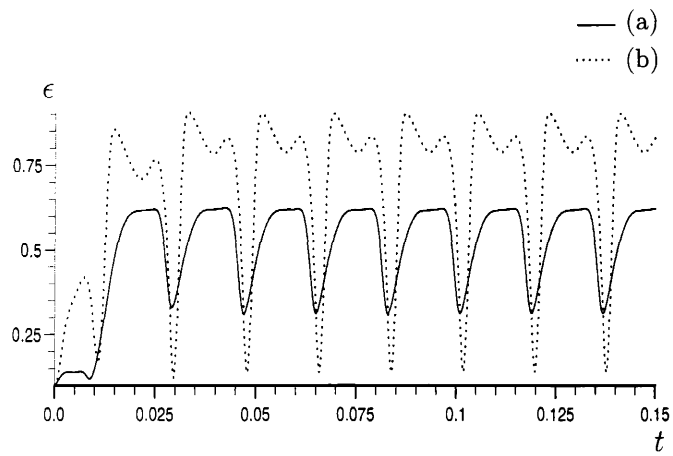

Figure 6: Plot of (a),(b) the eccentricity of the two journal paths shown in Figure 5a,b respectively.

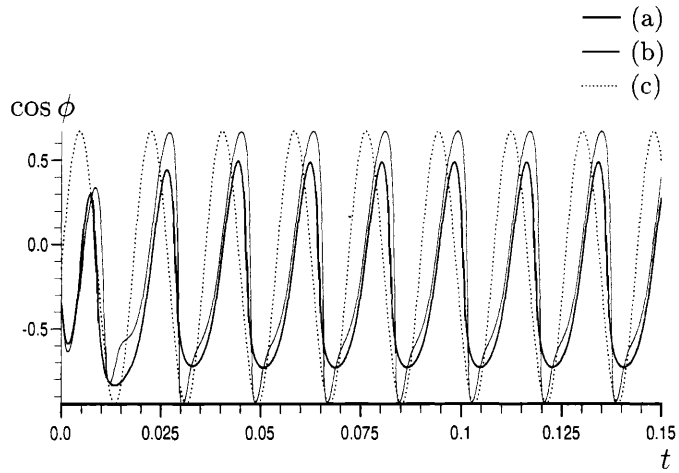

Figure 7: Plot of (a),(b) the cosine of the attitude angle of the journal paths shown in Figure 5a,b respectively together with (c) $k_1+k_2\sin(\omega t)$ for reference with the journal's angular velocity.

just its own weight. The initial position of the journal is specified by $\epsilon_o = 0.1$, and $\phi_o = 0.0$. The following parameters were used in the viscosity law (4): $\eta_o = 9.352\times10^{-4}$, $\eta_\infty = 4.5\times10^{-4}$, $\alpha = 1.119\times10^{-8}$, $\bar{\alpha} = 2.39\times10^{-8}$, $E = -13.527$, $F = 2.297$ and $m = 0.545$. These values are taken from experimental measurements of typical non-Newtonian lubricants. We assume that full-film conditions hold, i.e., the lubricant does not cavitate. The resulting path is shown in Fig.8. The corresponding evolution of the eccentricity and attitude angle are shown in Figs. 9 and 10, respectively. The journal moves from its initial position until it reaches an equilibrium point. At this point the weight of the journal balances the reaction force of the lubricant on the journal. This example shows that it is possible to obtain stable trajectories of the journal without the inclusion of a cavitation model. Variable viscosity is sufficient to produce a pressure field that is not exactly anti-symmetric and this is responsible for the stabilising effect. In the case of constant viscosity the pressure field is anti-symmetric and therefore without cavitation the bearing always fails. Further work needs to be done on the role of variable viscosity.

7 Concluding remarks

A moving spectral element method has been described for solving a problem of tremendous industrial importance. Some of the key modelling aspects have been described. The position of the journal has been tracked by solving an equation of motion for the journal. The preconditioner for the PCG method is chosen dynamically in time to ensure that the solution for the flow of the lubricant at each time step is performed efficiently and within a prescribed number of iterations.

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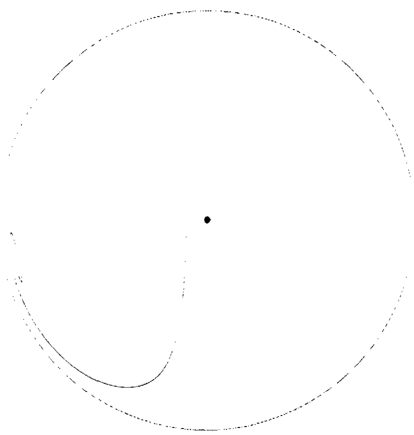

Figure 8: The path of journal with a full-film variable viscosity lubricant with $M_e = 7 \times 10^4$ kg/m, ω=250rad/s, (ϵ_0,ϕ_0)=(0.1,0.0), F_p=F_c=0 N/m with time $t \in [0,0.1]$. The viscosity parameters are given in the paper.

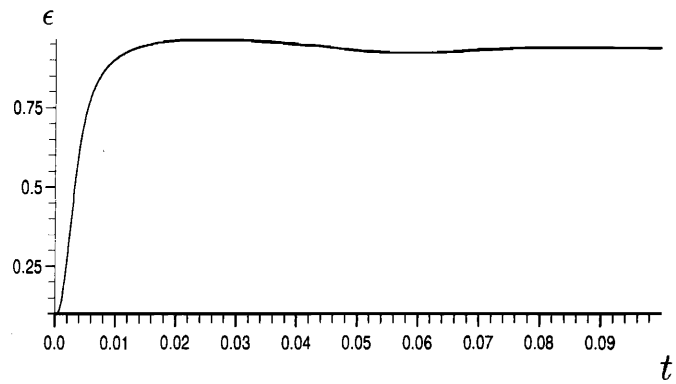

Figure 9: Plot of the eccentricity of the journal's path shown in Figure 8.

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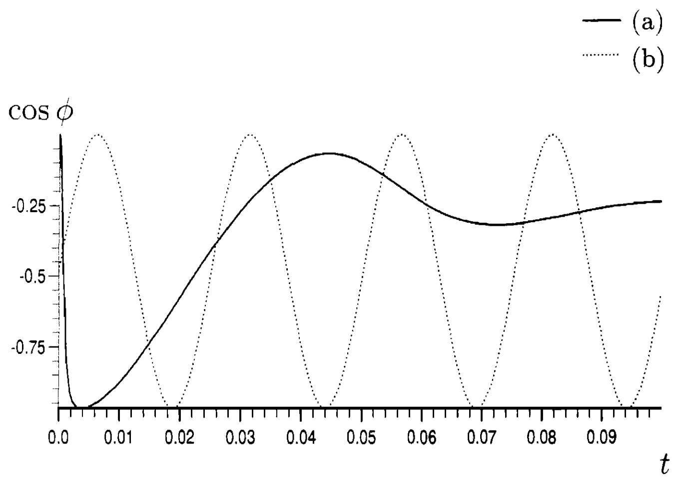

Figure 10: Plot of (a) the cosine of the attitude angle of the journals path shown in Figure 8 together with (b) $k_1 + k_2 \sin(\omega t)$ for reference with the journal's angular velocity.

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