Useful Finite Summation Identities ($a \neq 1$)

$$\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$
$$\sum_{k=0}^{n} ka^k = \frac{a}{(1-a)^2}[1-(n+1)a^n+na^{n+1}]$$
$$\sum_{k=0}^{n} k^2a^k = \frac{a}{(1-a)^3}[(1+a)-(n+1)^2a^n+(2n^2+2n-1)a^{n+1}-n^2a^{n+2}]$$

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{k=0}^{n} k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

Useful Infinite Summation Identities ($|a| < 1$)

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$
$$\sum_{k=0}^{\infty} ka^k = \frac{a}{(1-a)^2}$$
$$\sum_{k=0}^{\infty} k^2a^k = \frac{a^2+a}{(1-a)^3}$$