A **linear function** is a function which has a constant rate of change, i.e. slope. The slope is the amount of change in the function value when the independent variable increases by 1.

Suppose \( y = f(x) \) is a function of \( x \). Then:

\[
\text{slope } m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in function}}{\text{change in variable}}
\]

**Equations**

**Slope – Intercept Form**
- A linear function has formula \( y = f(x) = mx + b \).
- \( m \) is the slope of the line.
- The point \((0, b)\) is the vertical \((y)\) intercept.
- In practical terms, \( b \) represents the initial value of the output.

**Point – Slope Form**
- Suppose we know that a linear function has slope \( m \) and passes through the point \((x_1, y_1)\), then the equation of the line can be written as \( y - y_1 = m(x - x_1) \).
- From this equation, solving for \( y \) gives the equation of the linear function.

**Example 1:** Give the formula for the linear function described:

a. slope of 7 and \( y \)-intercept \((0, -2)\).

b. slope of \(-4\) and passes through the point \((2, -3)\).
c. passes through the points \((0, 4)\) and \((2, -6)\).

d. passes through the points \((-3, 5)\) and \((7, 24)\).

**Example 2:** Suppose that at the beginning of an experiment there are 500 bacteria present and that this number is decreasing at a rate of 75 bacteria per hour.

a. How can we tell that this relationship is linear?

b. Give a formula for \(N\), the number of bacteria after \(h\) hours.
Example 3: A certain company manufactures widgets. Suppose that the cost of leasing the building, buying the equipment, but producing no widgets is $14000. Suppose the total cost is $20000 if 500 widgets are produced.

a. Assuming a linear relationship between total cost $C$ and number of widgets produced $n$, find and interpret the slope of the function $C = f(n)$.

b. Give the formula for the function $C = f(n)$.

c. What is the total cost to when 785 widgets are produced.
Example 4: A certain jeweler makes a profit of $160 when she sells 12 necklaces and $300 when she sells 17 necklaces.

a. Assuming a linear relationship between profit $P$ and the number of necklaces sold $n$, find and interpret the slope of the function $P = f(n)$.

b. Give the formula for the function $P = f(n)$. 
