Recall, exponential functions of the form \( f(x) = P a^x \) has a fixed base \( a \), and the exponent varies. For a power function this is reversed. There is a fixed exponent, and the base varies.

**Power Functions**

For a power function \( f(x) = c x^k \)

- \( k \) is called the power and it is the most significant part of a power function.
- The coefficient \( c \) is equal to \( f(1) \).
- If \( k \) is positive, then \( f \) is increasing; larger, positive values of \( k \) cause \( f \) to increase more rapidly.
- If \( k \) is negative, then \( f \) decreases toward zero; negative values of \( k \) that are larger in size cause \( f \) to decrease more rapidly.

**Graphs of power functions**
Example 1: When a rock is dropped from a tall structure, it will fall $D = 16t^2$ feet in $t$ seconds.

a. Make a graph that shows the distance the rock falls versus time if the building is 70 feet tall.

b. How long does it take the rock to strike the ground?

Homogeneity Property of Power Functions

What happens to a power function when you double the variable? Triple the variable?

Example 2: The area $A$ of a square with side length $s$ is equal to $s^2$. Calculate the area of a square if the lengths of the sides are

a. Doubled.

b. Tripled.

c. Quadrupled

Example 3: The volume $V$ of a cube with side length $s$ is equal to $s^3$. Calculate the volume of a cube if the lengths of the sides are

a. Doubled.
b. Tripled.

c. Quadrupled

**General Rule:**
For a power function \( f(x) = cx^k \), if \( x \) is increased by a factor of \( t \), then \( f \) is increased by a factor of \( t^k \).

**Example 4:** The speed at which certain animals run is a power function of their stride length, and the power is \( k = 1.7 \). If one animal has a stride length three times as long as another, how much faster does it run?

**Example 5:** Let \( f(x) = cx^{2.53} \). By what factor must \( x \) be increased in order to triple the value of \( f \)?

**Example 6:** Let \( f(x) = cx^{1.47} \). If \( x \) is doubled in value, by what factor would \( f \) be increased?

**Example 7:** Let \( f(x) = cx^k \). Suppose \( f(6.6) \) is 6.2 times as large as \( f(x) = 1.76 \). What is the value of \( k \)?
Example 8: Let $f(x) = cx^{-1.32}$ and suppose $f(5) = 11$. Find the value of $c$.

Example 9: A biologist has determined that the weight of a certain fish is a power function of its length. He also has determined that when the length doubles, the weight increases by a factor of 7.4. What is the power $k$?

Comparing Exponential and Power Functions

Example 10: Let’s compare $f(x) = 2^x$ and $g(x) = x^2$. 
**Conclusion:** Over a sufficiently large horizontal span, an exponential function (with base larger than 1) will increase much more rapidly than a power function.