Math 1311  
Section 6.2  
Rates of Change for Other Functions

- Notation $\frac{df}{dx}$ means the rate of change in $f$ with respect to $x$.
- $\frac{df}{dx}$ tells how much $f$ is expected to change if $x$ is increased by 1 unit.
- $\frac{df}{dx}$ is a function of $x$.

**Example 1:** If $S = S(t)$ gives directed distance for an object as a function of time $t$, then
$$\frac{ds}{dt}$$

is the rate of change in directed distance with respect to time. This is ________________.

$\frac{ds}{dt}$ tells the additional distance we expect to travel in 1 unit of time.

We are currently located 100 miles south of Dallas, Texas, and traveling south with a velocity of 50 miles per hour, then in 1 additional hour we would expect to be _______ miles south of Dallas.

**Example 2:** If $V = V(t)$ is the velocity of an object as a function of time $t$, then
$$\frac{dv}{dt}$$

is the rate of change in velocity with respect to time. This is ________________.

$\frac{dv}{dt}$ tells the additional velocity we expect to attain in 1 unit of time.

We are traveling with a velocity 50 miles per hour, and if we start to pass a track our acceleration might be 2 miles per hour each second. Then 1 second in the future, we would expect our velocity to be ________ miles per hour.

**Example 3:** If $T = T(D)$ denote the amount of income tax, in dollars, that you pay on an income of $D$ dollars, then $\frac{dT}{dD}$ is the rate of change in tax with respect to the money you earn.

This is ________________.

$\frac{dT}{dD}$ tells us the additional tax you expect to pay if you earn 1 additional dollar.

Suppose we have a tax liability of $3000 and our marginal tax rate is 0.2.

If we earn an additional $1, we would expect our tax liability to increase by _______ to a total of ________;

If we earn an additional $100, we would expect our tax liability to increase by _______ to a total of ________.
**Example 4:** If $P = P(i)$ is the profit, in dollars, that you expect to earn on an investment of $i$ dollars, and then $\frac{dP}{di}$ is the rate of change in profit with respect to dollars invested.

$\frac{dP}{di}$ tells how much additional profit to be expected if 1 additional dollar is invested.

This is ____________________________________________.

If your current investment in a project gives a profit of $1000, and our marginal profit is 0.2, then we would expect that an additional investment of $100 dollars would give additional profit of ________ for a total profit of ________.

**Fundamental Properties of Rates of Change**

For a function $f = f(x)$ we will use the notation $\frac{df}{dx}$ to denote the rate of change in $f$ with respect to $x$.

1. The expression $\frac{df}{dx}$ tells us how $f$ changes in relation to $x$. It gives the additional value that is expected to be added to $f$ if $x$ increases by 1 unit.
2. When $f$ is increasing, $\frac{df}{dx}$ is positive.
3. When $f$ is decreasing, $\frac{df}{dx}$ is negative.
4. When $f$ is not changing, $\frac{df}{dx}$ is zero.
5. When $\frac{df}{dx}$ is constant, then $f$ is a linear function with slope $\frac{df}{dx}$.

**Example 5:** What is the common term for the rate of change of each of the following phenomena?

a) Directed distance as a function of time
b) Tax due as a function of income
c) Profit as a function of dollars invested
d) Velocity as a function of time

**Exemple 6:** Suppose $f = f(x)$. What is the sign of $\frac{df}{dx}$ in each of the following situations?

a) The function $f$ is increasing.
b) The function $f$ has reached a peak.
c) The function $f$ is decreasing.
d) The graph of $f$ is a horizontal line.
**Example 7:** Describe $f$ if $\frac{df}{dx} = 10$.

**Example 8:** What can be said about the graph of $f$ if the graph of $\frac{df}{dx}$ is below the horizontal axis?

**Example 9:** Let $s(a)$ denote sales generated by spending $a$ dollars on advertising. My goal is to increase sales. If $\frac{ds}{da}$ is negative, should I spend more or less money on advertising?

**Example 10:** The price $P$ of gasoline decreases to a minimum and starts to increase. What is the rate of change $\frac{dp}{dt}$ of the price with respect to time $t$ at the time when the price reaches a minimum?