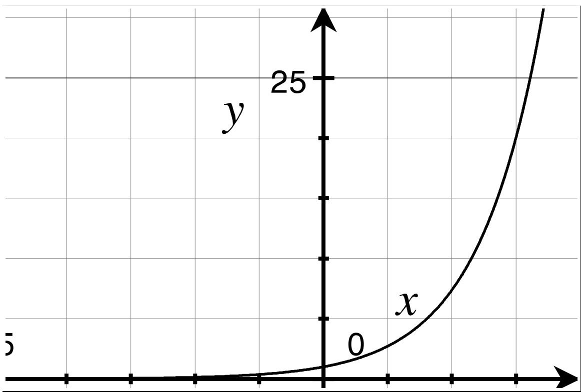
4.4 Modeling Nearly Exponential Data: Exponential Regression

As we saw in chapter 3, real data rarely fits any function perfectly. If the average rate of change between successive points is not constant, the data is not linear. However if the scatter plot of the data looked fairly linear, we could model the data with a linear regression.

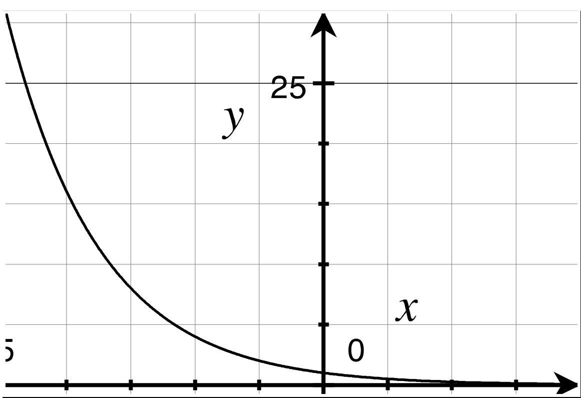
Similarly, if the growth or decay factor between successive points is not constant, the data is not exponential. But if the scatter plot shows the appropriate graphical properties, the data can be modeled with an exponential regression.

Exponential Function Graphs

The graph of an exponential function $f(x) = Pa^x$ has one of two forms (depending on whether it is exponential growth or decay).



a > 1: exponential growth



a < 1: exponential decay

The graphs of exponential functions are always concave up and either always increasing or always decreasing.

If the scatter plot of a data set shows either of these "shapes", then an exponential regression model is likely appropriate.

Example 1

The table below shows the U.S. population (measured in millions) for various years. Let x represent the number of years since 1800.

Year	1800	1810	1820	1830	1840	1850	1860
Population	5.31	7.24	9.64	12.87	17.07	23.19	31.44

(a) Look at a scatter plot of the data. Discuss the significant features and whether a linear or exponential model seems more appropriate.

(b) Find a regression model of this type.

(c) Estimate the U.S. population in 1836 and in 1900. Which of these is more likely to be accurate? Explain.

Example 2

Look at a scatterplot for each of the following functions. Determine if an exponential regression model is appropriate. If it is, find the model and state the constant percentage growth rate of the model. If not, explain why not.

(a)

x	2	5	6	9	10
y	4.2	6.4	7.4	11.3	12.9

(b)

x	5	7	9	11	13	15	17
y	76	75	79	87	92	95	92

(c)

x	7	8	11	13	14	15	19	22
y	36.3	32	22.25	18.25	17	16.25	18.25	25

(d)

x	1	2	3	4	5
y	0.7	0.3	0.1	0.05	0.01

Example 3

The relationship between length L in millimeters ad caloric content C in calories for a certain mollusk is shown in the table below.

L	7.5	13	20	24	31
C	92	200	625	1035	1950

(a) Look at a scatterplot of the data. Discuss the features that support that an exponential model is appropriate.

(b) Find an exponential regression for the data. By what percent is the caloric content increasing according to the model?