Section 5.3: Modeling Data with Power Functions

In this section we will investigate procedures for modeling data with a power function. First we will look at how to construct a power function from two points (by hand) and then we will look at power function regression from a table of data.

Power Function from Two Points

Given two points, to construct a power function passing through them:

- Using the ratio of the two points and logarithms, we can determine the power k
- Plugging one of the two points back in to $y = cx^k$, we can determine c

For Example:

Find the formula for the power function that passes through the points $\overset{x_1}{(2,}\ \overset{y_1}{12.8)}$ and $\overset{x_2}{(6,}\ \overset{y_2}{115.2)}$.

$$y = cx^k$$

$$y_1 = c\,2^k$$

$$y_2 = c\,6^k$$

$$\frac{y_2}{y_1} = \frac{c6^k}{c2^k} = \frac{f(6)}{f(2)} = \frac{115.2}{12.8} = 9$$

$$\frac{\cancel{c}6^k}{\cancel{c}2^k} = \left(\frac{6}{2}\right)^k$$

$$\Rightarrow \boxed{3^k = 9}$$

$$\Rightarrow k\log 3 = \log 9$$

$$\Rightarrow k = \frac{\log 9}{\log 3} = 2$$

$$\log 3^k = \log 9$$

Thus, $f(x) = cx^2$.

$$(\overset{x}{2},\ \overset{f(x)}{12.8})$$

Then:

$$c(2)^2 = 12.8$$

$$\Rightarrow c = \frac{12.8}{2^2} = 3.2$$

Thus, $f(x) = 3.2x^2$

Example 1

Find the formula for the power function passing through the given points.

(a) $(4, 38.4)$ & $(25, 600)$

$x \quad f(x)$

$$\frac{\cancel{c}\,25^k}{\cancel{c}\,4^k} = \frac{600}{38.4}$$

$$\log\left(\frac{25}{4}\right)^{k} = \log 15.625$$

$$k\log\left(\frac{25}{4}\right) = \log 15.625$$

$$k = \frac{\log 15.625}{\log(25/4)} = 1.5$$

$$f(x) = cx^{1.5}$$

$$\frac{38.4}{4^{1.5}} = \frac{c4^{1.5}}{4^{1.5}}$$

$$\frac{38.4}{4^{1.5}} = c$$

$$c = 4.8$$

$$\boxed{f(x) = 4.8x^{1.5}}$$

(b) $(2, 3.1)$ & $(8, 0.19375)$

$x \quad f(x)$

$$\frac{\cancel{c}\,8^k}{\cancel{c}\,2^k} = \frac{.19375}{3.1}$$

$$\left(\frac{8}{2}\right)^k = .0625$$

$$\log 4^{k} = \log .0625$$

$$k\log 4 = \log .0625$$

$$k = \frac{\log .0625}{\log 4}$$

$$k = -2$$

$$f(x) = cx^{-2}$$

$$\frac{3.1}{2^{-2}} = \frac{c\cancel{2^{-2}}}{\cancel{2^{-2}}}$$

$$c = \frac{3.1}{2^{-2}} = 12.4$$

$$\boxed{f(x) = 12.4x^{-2}}$$

(c) $(2, 136)$ & $(5, 5312.5)$

$x \quad f(x)$

$$\frac{\cancel{c}\,5^k}{\cancel{c}\,2^k} = \frac{5312.5}{136}$$

$$\left(\frac{5}{2}\right)^k = 39.0625$$

$$\log 2.5^{k} = \log 39.0625$$

$$k\log 2.5 = \log 39.0625$$

$$k = \frac{\log 39.0625}{\log 2.5} = 4$$

$$f(x) = cx^4$$

$$\frac{136}{2^4} = \frac{c2^4}{2^4}$$

$$c = \frac{136}{2^4} = 8.5$$

$$\boxed{f(x) = 8.5x^4}$$

(d) $(4, 189.4318)$ & $(10, 1708.08277)$

$x \quad f(x)$

$$\frac{\cancel{c}\,10^k}{\cancel{c}\,4^k} = \frac{1708.08277}{189.4318}$$

$$\left(\frac{10}{4}\right)^k =$$

$$\log(2.5)^{k} = \log$$

$$k\log(2.5) = \log$$

$$k = \frac{\log \frac{1708.08277}{189.4318}}{\log 2.5} = 2.4$$

$$f(x) = cx^{2.4}$$

$$\frac{189.4318}{4^{2.4}} = \frac{c4^{2.4}}{4^{2.4}}$$

$$c = 6.8$$

$$\boxed{f(x) = 6.8x^{2.4}}$$

Power Function Regressions

As with other function regression models we have studied, a table of data can be modeled with a power function regression model.

Example 2

A sailor records the distance D, in miles, to the visible horizon at several heights h, in feet above the surface of a calm ocean. His results are in the table below.

h	D
6	3.3
8	3.6
12	4.7
16	5.4
19	5.9

(a) Find the power regression model for this data.

(b) Use function notation to represent the distance to the visible horizon at a height of 14.2 feet.

(a) $D(h) = 1.2577x^{.5249}$

(b) $D(14.2) = 1.2577(14.2)^{.5249}$

$= 5.06$

Example 3

Population of cities and driving times are related, as shown in the table, which shows the 1960 population of several cities N, in thousands, with the average time T, in minutes spent by residents driving to work.

N	T
6489	16.8
1804	12.6
1808	14.3
38	6.1
347	10.8
48	7.3

(a) Find the power regression model for this data.

(b) Use function notation to represent the average commute time in a city of population half a million.

(a) $N(T) = 3.4025T^{.1843}$

(b) $N(500) = 10.7$

500,000