Math 1312
Section 1.1 : Sets, Statements, and Reasoning

A **set** is any _____.

These objects are called the _____ of the set.

A is a **subset** of B, if A is "contained" inside B. That is, all elements of A are also elements of B,

in symbols, A_____B.

NOTE: A and B may coincide, i.e. be the same (A = B).

A set with no elements is an _____ set, denoted _____.

Example 1: Examples of sets:
    A = {1, 2, 3}
    B = {Counting numbers }
    C= {even numbers less than 10}
    D = {Students enrolled in Math 1312}

Set A has 3 elements all of which are also the elements of B, i.e $A \subseteq B$.

Elements common to A and B form the **intersection** of A and B, written as _____

The **union** of two sets is all elements that are in A or B, written as _____

Example 2:

Find: $A \cap B$

Find: $A \cap C$

Find: $A \cup C$

Find: $B \cup D$

A **statement** is a set of _____ or _____ that collectively make a claim that can be classified as true or false.

Example 3

Classify the following as a true statement, false statement, or neither.

$5 + 4 = 9$ $5 < 2$ Triangles have 3 sides.

Texas is the largest state in US.   Watch out!

An _____ **statement** is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.

Example 4

a. $x + 2 = 5$ b. She is a good tennis player

.

The_____ of a statement P makes a claim opposite that of the original statement, written as $\sim P$.

Example 5
Statement: All fish can swim.

Negation:

Example 6

Write negations for the following statements. Determine the truth value of both, the statement and its negation.

1. Statement: A rectangle has 4 sides.
   Negation:

2. Statement: $2 + 6 = 8$
   Negation:

3. Statement: $5 \leq 2$ $5 \geq 2$ $5 > 2$ $5 < 2$
   Negation:

4. Statement: All jokes are funny.
   Negation:

NOTE: A statement and its negation have OPPOSITE truth values!

Construct a **truth table** for the negation of P.

P	$\sim P$

We form a **compound statement** by combining simple statements.
Let's use letters P and Q to represent two simple statements.

> ➢ **Conjunction:** P _____ Q
> ➢ **Disjunction:** P _____ Q ___

A conjunction is TRUE only if BOTH P and Q are true.

A disjunction is FALSE only if BOTH P and Q are false.

Complete the truth value tables for conjunction and disjunction of P and Q.

P	Q	P and Q
T	T	
T	F	
F	T	
F	F	

P	Q	P or Q
T	T	
T	F	
F	T	
F	F	

Example 7:

Decide if the statement is a Conjunction or Disjunction? Then decide if statement is True or False?

1. Triangles are square or circles are round.

2. Triangles are round or circles are square.

3. $2 < 1$ and $5 < 7$

4. Triangles have 3 angles and $2 + 3 = 5$.

Conditional statement is a compound statement "If P, than Q". Here, P is called the

_____and Q is called the _____.

"If P, then Q" can be expressed in the form "All P are Q".

Example 8

1. If an animal is a fish, then it can swim.

2. If a student is enrolled in this class, then she has to pay the tuition.

Example 9
State the <mark>hypothesis</mark> and the <mark>conclusion</mark>.

1. All squares are rectangles.

2. You get an "A" in Math 1312 class if you study hard.

Conditional statement is FALSE only if hypothesis is TRUE but conclusion is FALSE.

P	Q	If P, then Q
T	T	T
T	F	F
F	T	T
F	F	T

Example 10
True or False?

1. $2 + 5 = 7$ if triangle has 4 angles.

2. If a square is round, then a circle is round.

3. If Tom studies, then he will get an A on the test.

4. If the Moon is made of cheese, then I am the queen of France.

_____is a process based on experience and principles that allow one to arrive at a conclusion.

Types of reasoning
1. **Induction** – is a way of reasoning that draws conclusions from a small number of observations.
2. **Deduction** – is a formal argument that proves the tested theory.

Example 11:

In a geometry class, you measured the three interior angles of 10 triangles. The triangle s all had 2 equal sides. You discovered that they all have two angles equal in measure.

1. What may you conclude?

2. What type of reasoning are you using?

Example 13.

If a student gets 95 in a test, then he gets an "A". Tom got 95 in the test.

1. If you accept the above statements as true, what must you conclude?

2. What type of reasoning are you using?

Law of detachment: Let P and Q represent simple statements and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form:

      1. If P, then Q
      2. P
      Conclusion: $\therefore$ Q

($\therefore$ - symbol that means _____ _)

    ➢ Allows drawing logic conclusions
    ➢ Can check if an argument is valid

Example 14
    Is the following argument valid?

      1. If it is raining, then Tom will stay at home.
      2. It is raining
      Conclusion: $\therefore$ Tom will stay at home.

      1. If a man lives in Houston, then he lives in Texas.
      2. Mark lives in Texas.
      Conclusion: $\therefore$ Mark lives in Houston.
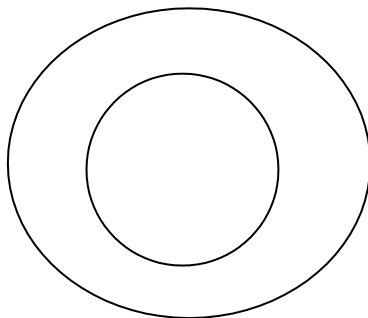

Example 15
    Use deduction to state a conclusion (if possible).

      1. If an angle is a right angle., then it measures 90 °.
      2. Angle C is a right angle.
      Conclusion: $\therefore$

Venn Diagrams
We often use Venn Diagrams to represent sets of objects.
"If P, then Q" can be represented as:

Example 15: *Use Venn Diagram to verify*
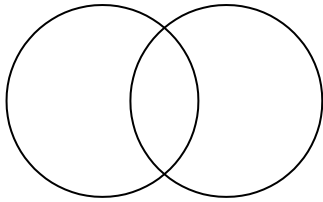
1. If a student gets 95 in a test, then he gets an A.
2. Tom got 95 in the test.
Conclusion: Tom got an A in the test.

A = {Students who got an A on the test}
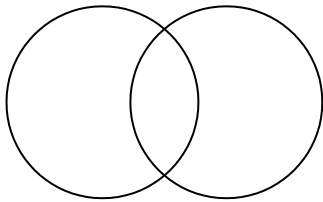B = {Students who got 95 on the test}

Note: We can also use Venn diagram to represent conjunction and disjunction:

The intersection of P and Q:

$P \cap Q$

The elements that are BOTH in P and in Q

The union of P and Q:

$P \cup Q$

The elements that are in P OR in Q

Example 16: *Use deduction to state a conclusion (if possible)*

1. If a person attends a university, then he will be a success in life.
2. Sam attends University of Houston.
Conclusion:

1. If the sum of the measures of two angles is 90^0, then they are called complementary
2. Angle 1 measures 50 degrees and angle 2 measures 40 degrees.
Conclusion:

Example 17: *Use Venn diagrams to determine whether the argument is valid or not*

1. If an animal is a cat, then it makes "meow" sound.
2. Tom is a cat.
Conclusion: ∴ Tom makes "meow" sound.

1. If an animal is a cat, then it makes "meow" sound.
2. Tom makes "meow" sound.
Conclusion: ∴ Tom is a cat.