

Read ahead and use your textbook to fill in the blanks. We will work the examples together.

A set is any collection of objects.

These objects are called the elements of the set.

A is a **subset** of B , if A is "contained" inside B . That is, all elements of A are also elements of B , in symbols, $A \subseteq B$.

NOTE: A and B may coincide, i.e. be the same ($A = B$).

A set with no elements is an empty set, denoted \emptyset .

Example 1: Examples of sets:

$$A = \{1, 2, 3\}$$

$$B = \{\text{Counting numbers}\}$$

$$C = \{\text{even numbers less than 10}\} = \{2, 4, 6, 8\}$$

$$D = \{\text{Students enrolled in Math 1312}\}$$

Set A has 3 elements all of which are also the elements of B , i.e. $A \subseteq B$.

Elements **common** to A and B form the **intersection** of A and B , written as $A \cap B$

The **union** of two sets is all elements that are in A or B , written as $A \cup B$

Example 2:

$$\text{Find: } A \cap B = \{1, 2, 3\}$$

$$\text{Find: } A \cap C = \{2\}$$

$$\text{Find: } A \cup C = \{1, 2, 3, 4, 6, 8\}$$

$$\text{Find: } B \cap D = \emptyset$$

A **statement** is a set of words or symbols that collectively make a claim that can be classified as true or false.

Example 3

Classify the following as a true statement, false statement, or neither.

$5 + 4 = 9$ **True** $5 < 2$ **False** Triangles have 3 sides. **True**

Texas is the largest state in US. **False** Watch out! **NOT a statement.**

An open **statement** is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.

Example 4

a. $x + 2 = 5$

if $x = 3$, true
if $x = 5$, false

b. She is a good tennis player

She = Ms. Irina, false
She = Serena Williams, true

The negation of a statement P makes a claim opposite that of the original statement, written as $\sim P$.

Example 5

Statement: All fish can swim.

True All \leftrightarrow Some

Negation:

Some fish cannot swim. **False**

Example 6

Write negations for the following statements. Determine the truth value of both, the statement and its negation.

1. Statement: A rectangle has 4 sides. True
Negation: A rectangle does not have 4 sides. False
2. Statement: $2 + 6 = 8$ True
Negation: $2 + 6 \neq 8$ False
3. Statement: $5 \leq 2$ F $5 \geq 2$ T $5 > 2$ T $5 < 2$ F
Negation: $5 > 2$ T $5 < 2$ F $5 \leq 2$ F $5 \geq 2$ T
4. Statement: All jokes are funny. 3 \leq 3 T
Negation: Some jokes are not funny.

NOTE: A statement and its negation have OPPOSITE truth values!

Construct a **truth table** for the negation of P .

P	$\sim P$
T	F
F	T

We form a **compound statement** by combining simple statements.
Let's use letters P and Q to represent two simple statements.

- **Conjunction:** P and Q
- **Disjunction:** P or Q



A conjunction is **TRUE** only if **BOTH** P and Q are **true**.

A disjunction is **FALSE** only if **BOTH** P and Q are **false**.

Complete the truth value tables for conjunction and disjunction of P and Q .

P	Q	$P \text{ and } Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \text{ or } Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 7:

Decide if the statement is a Conjunction or Disjunction? Then decide if statement is True or False?

1. $\overset{F}{\text{Triangles are square}} \text{ or } \overset{T}{\text{circles are round.}}$ Disj., True
2. $\overset{F}{\text{Triangles are round}} \text{ or } \overset{F}{\text{circles are square.}}$ Disj., False
3. $\overset{F}{2 < 1} \text{ and } \overset{T}{5 < 7}$ Conj., False
4. $\overset{T}{\text{Triangles have 3 angles}} \text{ and } \overset{T}{2 + 3 = 5.}$ Conj., True

Conditional statement is a compound statement “If P , then Q ”. Here, P is called the

hypothesis and Q is called the conclusion.

“If P , then Q ” can be expressed in the form “All P are Q ”.

Example 8

1. $\overset{P}{\text{If an animal is a fish,}} \text{ then } \overset{Q}{\text{it can swim.}}$
 $\text{All fish } P \text{ can swim. } Q$
2. $\overset{P}{\text{If a student is enrolled in this class,}} \text{ then } \overset{Q}{\text{she has to pay the tuition.}}$

$\text{All students enrolled in this class pay the tuition.}$

Example 9

State the hypothesis and the conclusion.

1. All squares are rectangles.

If a shape is a square, then it is a rectangle.

2. You get an "A" in Math 1312 class if you study hard.

! If you study hard, then you get an "A" in Math 1312 class.

Conditional statement is FALSE only if hypothesis is TRUE but conclusion is FALSE.

P	Q	If P , then Q
T	T	T
T	F	F
F	T	T
F	F	T

contract

Example 10

True or False?

1. $2 + 5 = 7$ if triangle has 4 angles.

If triangle has 4 angles, then $2+5=7$.
F T True

2. If a square is round, then a circle is round.

F T True

3. If Tom studies, then he will get an A on the test.

contract broken

4. If the Moon is made of cheese, then I am the queen of France.

F F True

Reasoning

is a process based on experience and principles that allow one to arrive at a conclusion.

Types of reasoning

1. **Induction** – is a way of reasoning that draws conclusions from a small number of observations.
2. **Deduction** – is a formal argument that proves the tested theory.

Example 11:

In a geometry class, you measured the three interior angles of 10 triangles. The triangles all had 2 equal sides. You discovered that they all have two angles equal in measure.

1. What may you conclude?

Two equal sides in a \triangle mean two equal angles.

2. What type of reasoning are you using?

Inductive

Example 13.

If a student gets 95 in a test, then he gets an "A". Tom got 95 in the test.

1. If you accept the above statements as true, what must you conclude?

Tom got an 'A'.

2. What type of reasoning are you using?

Deductive

Law of detachment: Let P and Q represent simple statements and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form:

1. If P , then Q True
2. P True
- Conclusion: $\therefore Q$

(\therefore - symbol that means therefore)

- Allows drawing logic conclusions
- Can check if an argument is valid

Example 14

Is the following argument valid?

1. If P it is raining, then Q Tom will stay at home.
2. P It is raining
- Conclusion: $\therefore Q$ Tom will stay at home.

1. If P , then Q .
2. P
 $\therefore Q$ valid

1. If P a man lives in Houston, then Q he lives in Texas.
2. P Mark lives in Texas.
- Conclusion: $\therefore Q$ Mark lives in Houston.

1. If P , then Q
2. Q
 $\therefore P$ invalid

Example 15

Use deduction to state a conclusion (if possible).

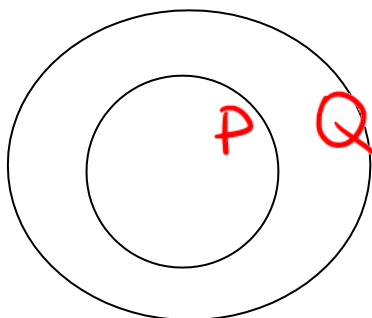
1. If P an angle is a right angle., then Q it measures 90° .
2. P Angle C is a right angle.
- Conclusion: \therefore

Angle C measures 90° .

Venn Diagrams

We often use Venn Diagrams to represent sets of objects.

"If P , then Q " can be represented as:



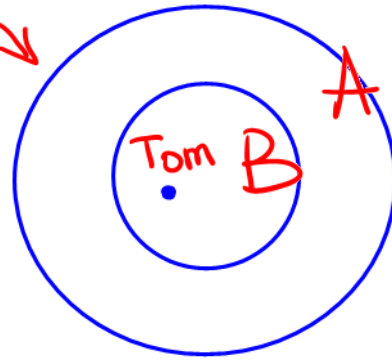
Example 15: Use Venn Diagram to verify

1. If a student gets 95 in a test, then he gets an A.
 2. Tom got 95 in the test.
- Conclusion: Tom got an A in the test.

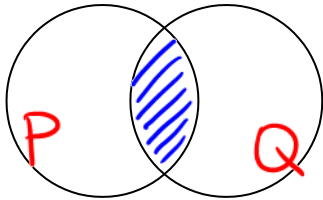
$A = \{\text{Students who got an A on the test}\}$

$B = \{\text{Students who got 95 on the test}\}$

valid



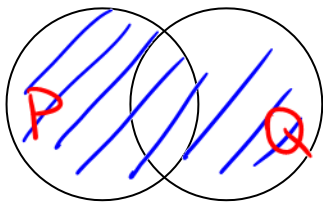
Note: We can also use Venn diagram to represent conjunction and disjunction:



The intersection of P and Q :

$P \cap Q$ (Conj)

The elements that are BOTH in P and in Q



The union of P and Q :

$P \cup Q$ (Disj)

The elements that are in P OR in Q

Example 16: Use deduction to state a conclusion (if possible)

1. If a person attends a university, then he will be a success in life.
2. Sam attends University of Houston.

Conclusion:

Sam will be a success in life.

1. If P , then Q
2. P
 $\therefore Q$

1. If the sum of the measures of two angles is 90° , then they are called complementary
2. Angle 1 measures 50 degrees and angle 2 measures 40 degrees.

Conclusion:

Angle 1 & Angle 2 are complementary.

Example 17: Use Venn diagrams to determine whether the argument is valid or not

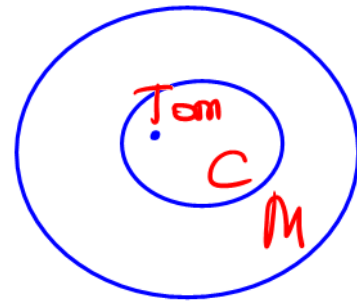
- C M
1. If an animal is a cat, then it makes "meow" sound.
 2. Tom is a cat.

Conclusion: \therefore Tom makes "meow" sound.

$C = \{\text{cats}\}$

$M = \{\text{animal meows}\}$

valid



1. If an animal is a cat, then it makes "meow" sound.
2. Tom makes "meow" sound.

Conclusion: \therefore Tom is a cat.

Invalid

