Math 1312
Sections 1.2, 1.3, and 1.4
Informal Geometry and Measurement; Early Definitions and Postulates; Angles and Their Relationships

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Undefined Terms (set, point, line, plane)

- A point, which is represented as a dot, has location but not size.

  We use upper case letters to name points.

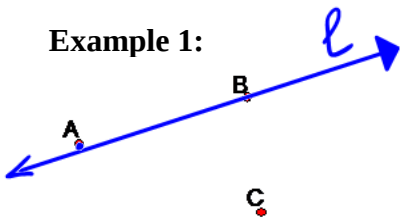
- A line is an infinite set of points. Notation: $\overleftrightarrow{AB}$

  Given any 3 distinct points on the same line, they are said to be **collinear**.

  Notation: $A - X - B$

$\overleftrightarrow{AB}$; $\overleftrightarrow{BA}$; ℓ

Example 1:



D

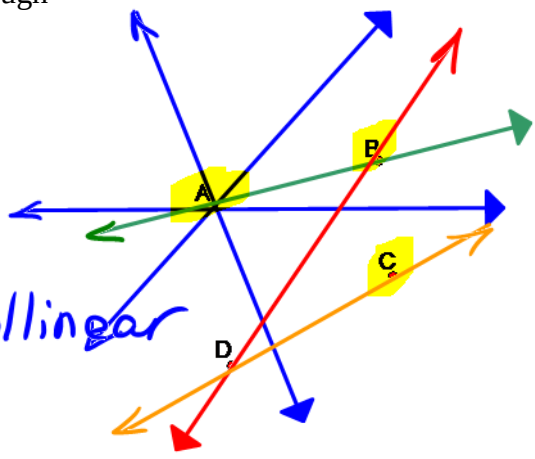
A Postulate is an unproved assumption.

Postulate: Through two distinct points there is exactly one line

Postulate: If two lines intersect, then they intersect at a point.

Example: How many lines can be drawn through

1. point *A*? ∞ inf many
2. both points *A* and *B*? one
3. all three points *A*, and *B*, and *C* ? none, points are
4. points *A*, and *B*, and *C* ? none collinear
5. Where do $\overleftrightarrow{BD}$ and $\overleftrightarrow{CD}$ intersect?



$\overleftrightarrow{AB}$
$\overleftrightarrow{BC}$
$\overleftrightarrow{AC}$

$\overleftrightarrow{BD} \cap \overleftrightarrow{CD} = D$

A segment is part of a line. It consists of two distinct points and all points between them.

Notation: $\overline{AB}$, where A and B are the endpoints.

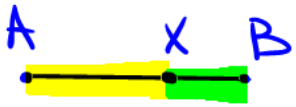
$\overline{BA}$

To **measure** segments we use rulers. Remember that there is a margin of error each time we use such a device.

Ruler Postulate: The measure of any line segment is a unique positive number.

Segments that have the same length are called congruent. Notation: $\overline{AB} \cong \overline{CD}$

In diagrams, we use identical tick marks to indicate congruent segments.

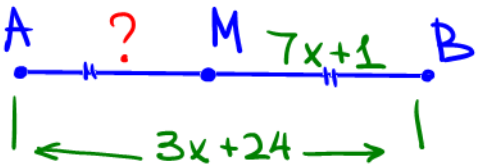


Segment-Addition Postulate: If X is a point of $\overline{AB}$ and $A - X - B$, then $AX + XB = AB$.

Definition: The midpoint of a line segment is the point that separates the line segment into two congruent parts.

Theorem: The midpoint of a line segment is unique.

Example: M is the midpoint of the segment AB. $AB = 3x + 24$ and $MB = 7x + 1$. Find x and the length of the segment AM.



$AB = 2MB$

$3x + 24 = 2(7x + 1)$

$3x + 24 = 14x + 2$

$24 = 11x + 2$

$22 = 11x$

$x = 2$

$AM = BM$

$= 7(2) + 1$

$= 15$

Example: M is the midpoint of the segment AB. $AM = 3x + 4$ and $MB = x + 38$. Find x and the length of the segment AB.

$AM = MB$

$3x + 4 = x + 38$

$2x + 4 = 38$

$2x = 34$

$x = 17$

$AB = 2(AM)$

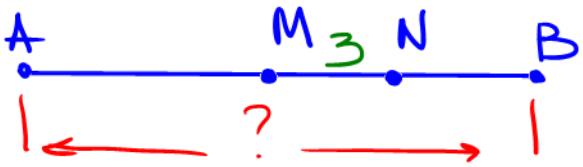
$AB = 2(3(17) + 4)$

$= 2(51 + 4)$

$= 2(55)$

$= 110$

Example:
M is the midpoint of the segment AB and N is the midpoint of the segment BM. Find AB, if $MN = 3$.



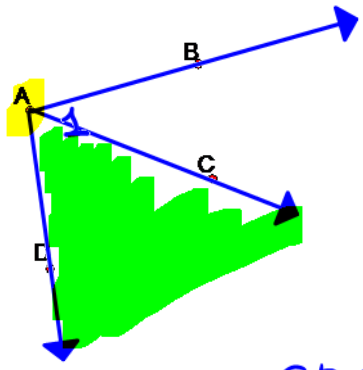
$AB = 2(BM)$
$= 2(2MN) = 4MN$
$= 4(3)$
$= 12$

Definition: A ray is made up of points and is straight. It begins at an endpoint and extends infinitely in one direction. Notation: $\overrightarrow{AB}$

Order!

Opposite rays are two rays that share a common endpoint and their union is a straight line.

Example: Draw rays $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Are they opposite rays? NO, do not form a line



$\angle BAD$;
$\angle DAB$;
$\angle A$; $\angle 1$

$\angle CAD$
$\angle DAC$
~~$\angle A$~~

Definition: An angle is union of two rays that share a common endpoint.
Notation: $\angle ABC$

The common point is called the vertex of the angle.

The rays are called sides of the angle.

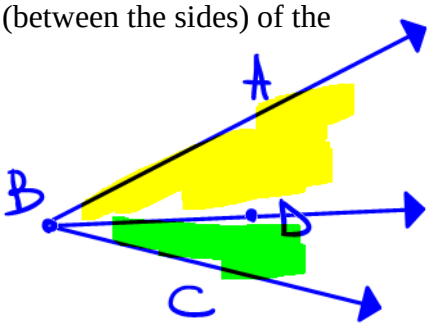
To **measure** angles we use a **protractor**. An angle's measure does not depend on the lengths of its sides.
Angles are measured in **degrees.** If the measure of an angle is 90. We write $m\angle ABC = 90^\circ$

Congruent angles are angles that have the same measure.

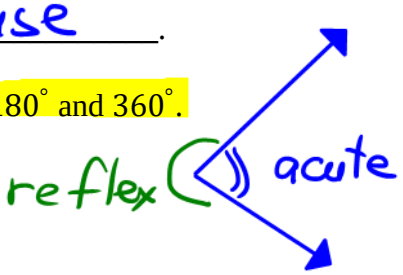
Example: $\angle ABC \cong \angle PQR$ means $m\angle ABC = m\angle PQR$

Angle - Addition Postulate: If a point D lies in the interior (between the sides) of the angle $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.

Draw a figure



- An angle whose measures less than 90° is an acute.
- An angle whose measures exactly 90° is a right.
- An angle whose measures exactly 180° is a straight .
- If an angle measures between 90° and 180° it is an obtuse.
- A reflex is one whose measure is between 180° and 360°.



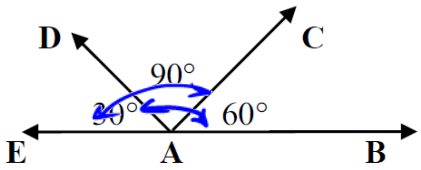
Example: Use the following figure to find

a) Straight angle $\angle BAE$

b) Right angle $\angle CAD$

c) Acute angle $\angle DAE, \angle CAB$

d) Obtuse angle $\angle DAB, \angle CAE$

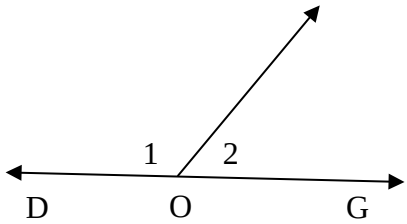


Example: $\angle DOG$ is a straight angle. If the $m\angle 2 = 65^\circ$ what is $m\angle 1$?

180°

$m\angle 1 = 180 - m\angle 2$

$= 180 - 65 = 115^\circ$



Definitions:

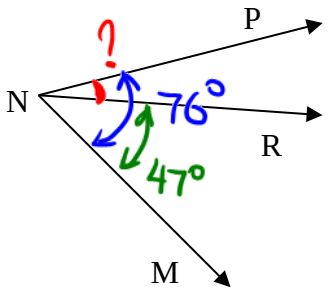
- The angle bisector of an angle is the ray that separates the given angle into two congruent angles.
- Two angles are complementary **angles** if the sum of their measures is 90°. Each angle in the pair is known as the **complement** of the other angle.
- Two angles are supplementary **angles** if the sum of their measures is 180°. Each angle in the pair is known as the **supplement** of the other angle.

Theorem: There is one and only one angle bisector for any given angle.

$\overrightarrow{BD}$ bisects $\angle ABC$

Examples:

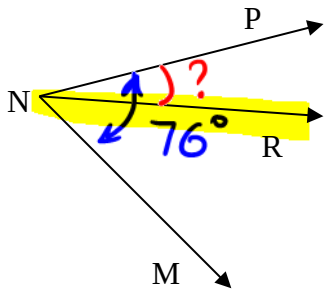
a) If $m\angle MNP = 76^\circ$ and $m\angle MNR = 47^\circ$, find $m\angle PNR$.



$m\angle PNR = m\angle MNP - m\angle MNR$

$= 76 - 47 = \boxed{29^\circ}$

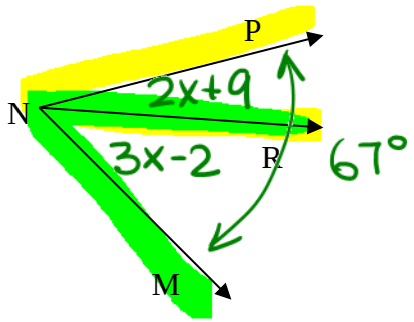
b) If $m\angle MNP = 76^\circ$ and $\overrightarrow{NR}$ bisects $\angle MNP$, find $m\angle PNR$.



$m\angle PNR = \frac{1}{2} m\angle MNP$

$= \frac{1}{2}(76) = \boxed{38^\circ}$

c) Find x, if $m\angle PNR = 2x + 9$ and $m\angle RNM = 3x - 2$, and $m\angle PNM = 67^\circ$.



Angle - Addition Postulate

$2x + 9 + 3x - 2 = 67$

$5x + 7 = 67$

$5x = 60$

$\boxed{x = 12}$

More Examples:

a) If $m\angle A = (2x)^\circ$, and $m\angle B = (x - 6)^\circ$, and $\angle A$ and $\angle B$ are complementary, find x and the measure of each angle.

$2x + x - 6 = 90$

$3x - 6 = 90$

$3x = 96$

$\boxed{x = 32}$

$m\angle A = 2(32) = \boxed{64^\circ}$

$m\angle B = 32 - 6 = \boxed{26^\circ}$

$m\angle B = 90 - 64 = 26^\circ$

b) If $m\angle A = (2y - 9)^\circ$ and $m\angle B = (7y)^\circ$ ~~the m $\angle B = (7y)^\circ$~~ $\angle A$ and $\angle B$ are supplementary, find y and the measure of each angle.

180°

$2y - 9 + 7y = 180$

$9y - 9 = 180$

$9y = 189$

$y = 21$

$m\angle A = 2(21) - 9 = 33^\circ$

$m\angle B = 7(21) = 147^\circ$

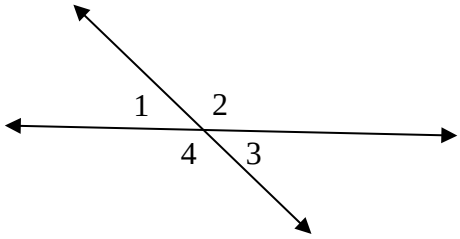
$m\angle B = 180 - 33 = 147^\circ$

Definitions:

- The two angles are adjacent if they have a common vertex and a common side between them.
- When two straight lines intersect, the pairs of nonadjacent angles in opposite positions are known as vertical **angles**.

Example: List all the pairs of adjacent and vertical angles.

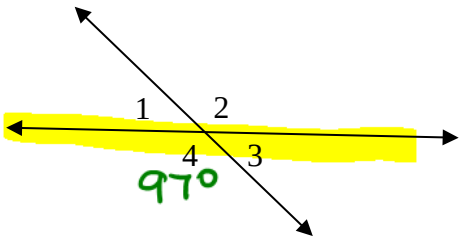
Adjacent	Vertical
$\angle 1$ & $\angle 2$	$\angle 1$ & $\angle 3$
$\angle 1$ & $\angle 4$	$\angle 2$ & $\angle 4$
$\angle 2$ & $\angle 3$	
$\angle 3$ & $\angle 4$	



Theorem: Vertical angles are congruent.

Examples:

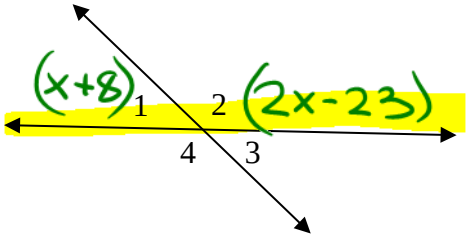
a) If $m\angle 4 = 97^\circ$, find the measures of all other angles.



$m\angle 2 = m\angle 4 = 97^\circ$

$m\angle 1 = m\angle 3 = 180 - 97 = 83^\circ$

b) If $m\angle 1 = (x + 8)^\circ$ and $m\angle 2 = (2x - 23)^\circ$, find x and the measures of all four angles.



$x+8+2x-23 = 180$

$3x-15 = 180$

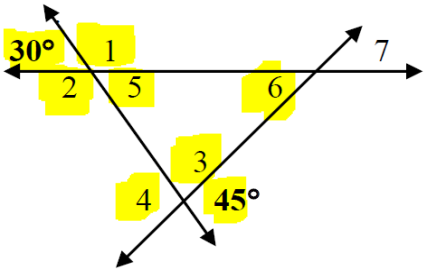
$3x = 195$

$x = 65$

$m\angle 1 = m\angle 3 = 65+8 = \boxed{73^\circ}$

$m\angle 2 = m\angle 4 = 2(65)-23 = 130-23 = \boxed{107^\circ}$

Example: Use the figure to find the measure of all the angles 1 -7.
Hint: $m\angle 3 + m\angle 5 + m\angle 6 = 180^\circ$.



$m\angle 5 = 30^\circ$

$m\angle 4 = 45^\circ$

$m\angle 1 = 180 - 30 = 150^\circ$

$m\angle 2 = 150^\circ$

$m\angle 3 = 180 - 45 = 135^\circ$

$m\angle 6 = 180 - 135 - 30 = 15^\circ$

$m\angle 7 = 15^\circ$