Definition (Merriam Webster): Proof is the process of establishing the validity of a statement.

We consider two column proofs.

PROOF	
Statements	**Reasons**
What?	Why?

In our proofs we can use the following properties.

Properties of Equality (a, b, and c are real numbers)	
Addition Property of Equality:	If $a = b$, then $a + c = b + c$.
Subtraction property of Equality:	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality:	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality:	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1: Which property of equality justifies each conclusion?

a. If $x + 2 = 10$, then $x = 8$.

b. If $\frac{2}{3}x = 8$, then $x = 12$

| Further Algebraic Properties of Equality (a, b, and c are real numbers)	
Reflexive Property:	$a = a$.
Symmetric Property:	If $a = b$, then $b = a$.
Distributive Property:	$a(b + c) = a \cdot b + a \cdot c$.
Substitution Property:	If $a = b$, then a replaces b in any equation.
Transitive Property:	If $a = b$ and $b = c$, then $a = c$.

Example 2: Given $3x + 2 = 4 + 5x$, prove $x = -1$.

PROOF

Statements	Reasons
1. $3x + 2 = 4 + 5x$	1.
2. $3x + 2 - 4 = 4 - 4 + 5x$	2.
3. $3x - 2 = 5x$	3.
4. $3x - 3x - 2 = 5x - 3x$	4.
5. $-2 = 2x$	5.
6. $\frac{1}{2}(-2) = \left(\frac{1}{2}\right)2x$	6.
7. $-1 = x$	7.
8. $x = -1$	8.

Example 3:
GIVEN: B is the midpoint of the segment $\overline{AC}$
PROVE: $AB = AC/2$

A ———————————————— C
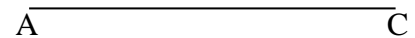
PROOF

Statements	Reasons
1. B is the midpoint of $\overline{AC}$	1.
2. $AB = BC$	2.
3. $AB + BC = AC$	3.
4. $AB + AB = AC$	4.
5. $2\,(AB) = AC$	5.
6. $AB = \frac{AC}{2}$	6.

Properties of Inequality (a, b, and c are real numbers)	
Addition Property of Inequality:	If $a > b$, then $a + c > b + c$. If $a < b$, then $a + c < b + c$.
Subtraction property of Inequality:	If $a > b$, then $a - c > b - c$. If $a < b$, then $a - c < b - c$.

Example 4:
GIVEN: $MN > PQ$
PROVE: $MP > NQ$

$\overline{\text{M} \qquad\qquad\qquad \text{N} \quad\;\; \text{P} \;\; \text{Q}}$

PROOF	
Statements	**Reasons**
1.	**1.**
2. $MN + NP > NP + PQ$	**2.**
3. $MN + NP = MP$ and $NP + PQ = NQ$	**3.**
4.	**4.** Substitution

Example 5: State the property or definition that justifies the conclusion.

Given that $\angle s$ 1 and 2 are complementary, then $m\angle 1 + m\angle 2 = 90°$.

Example 6: Draw a conclusion based on the stated property or definition.

a. Given: $m\angle 1 + m\angle 2 = 180°$; definition of supplementary angles.

Study more examples from the textbook!

b. Given: K is in the interior of $\angle GHJ$; Angle-Addition Postulate.

c. Given: $\frac{1}{2} = 0.5$ and $0.5 = 50\%$; Transitive Property of Equality