Math 1312
Section 1.5
Introduction to Geometric Proof

Definition (Merriam -Webster): Proof is the process of establishing the validity of a statement.

We consider two column proofs.

PROOF	
Statements	**Reasons**
What?	Why?

In our proofs, we can use the following properties.

$5 = 5$
$5-2 = 5-2$
$5+(-2) = 5+(-2)$

$\frac{a}{2} = \frac{1}{2}a$

Properties of Equality (a, b, and c are real numbers)	
Addition Property of Equality:	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality:	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality:	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality:	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1: Which property of equality justifies each conclusion?

a. If $x + \cancel{2} = 10$, then $x = 8$.
$\quad\;\; -\cancel{2} \quad -2$

Subtraction Prop. of Equality

b. If $\frac{2}{3}x = 8$, then $x = 12$

$\left(\frac{\cancel{3}}{\cancel{2}}\right)\frac{\cancel{2}}{\cancel{3}}x = 8\left(\frac{3}{2}\right)$

Mult. Prop. of Eq.

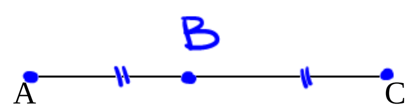
Further Algebraic Properties of Equality (a, b, and c are real numbers)	
Reflexive Property:	$a = a$.
Symmetric Property:	If $a = b$, then $b = a$.
Distributive Property:	$a(b + c) = a \cdot b + a \cdot c$.
Substitution Property:	If $a = b$, then a replaces b in any equation.
Transitive Property:	If $a = b$ and $b = c$, then $a = c$.

Example 2: Given $3x + 2 = 4 + 5x$, prove $x = -1$.

PROOF

Statements	Reasons
1. $3x + 2 = 4 + 5x$	**1.** Given
2. $3x + 2 - 4 = 4 - 4 + 5x$	**2.** Subt. Prop. of Equality.
3. $3x - 2 = 5x$	**3.** Substitution
4. $3x - 3x - 2 = 5x - 3x$	**4.** Subt. Prop. of Equality
5. $-2 = 2x$	**5.** Substitution
6. $\frac{1}{2}(-2) = \left(\frac{1}{2}\right)2x$	**6.** Mult. Prop. of Equal.
7. $-1 = x$	**7.** Substitution
8. $x = -1$	**8.** Symmetric

Example 3:
GIVEN: B is the midpoint of the segment $\overline{AC}$
PROVE: $AB = AC/2$

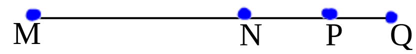


PROOF

Statements	Reasons
1. B is the midpoint of $\overline{AC}$	**1.** Given
2. $AB = BC$	**2.** Def. of the midpoint
3. $AB + BC = AC$	**3.** Segment-Add. Postulate
4. $AB + AB = AC$	**4.** Substitution
5. $2(AB) = AC$	**5.** Substitution
6. $AB = \frac{AC}{2}$	**6.** Div. Prop. of Eq.

Properties of Inequality (a, b, and c are real numbers)	
Addition Property of Inequality:	If $a > b$, then $a + c > b + c$. If $a < b$, then $a + c < b + c$.
Subtraction property of Inequality:	If $a > b$, then $a - c > b - c$. If $a < b$, then $a - c < b - c$.

Example 4:
GIVEN: $MN > PQ$
PROVE: $MP > NQ$



PROOF	
Statements	**Reasons**
1. $MN > PQ$	**1.** Given
2. $MN + NP > NP + PQ$	**2.** Add. Prop. of Ineq.
3. $MN + NP = MP$ and $NP + PQ = NQ$	**3.** Segment - Add. Postulate
4. $MP > NQ$	**4.** Substitution

Example 5: State the property or definition that justifies the conclusion.

Given that $\angle s$ 1 and 2 are complementary, then $m\angle 1 + m\angle 2 = 90^{\circ}$.

Def of comp. $\angle s$

Example 6: Draw a conclusion based on the stated property or definition.

Study more examples from the textbook!

a. Given: $m\angle 1 + m\angle 2 = 180^{\circ}$; definition of supplementary angles.

$\angle 1$ & $\angle 2$ are supplementary

b. Given: K is in the interior of $\angle GHJ$; Angle-Addition Postulate.

$m\angle KHG + m\angle KHJ = m\angle GHJ$

c. Given: $\frac{1}{2} = 0.5$ and $0.5 = 50\%$; Transitive Property of Equality.

$\frac{1}{2} = 50\%$