Math 1312
Section 1.6
Relationships: Perpendicular Lines

A **plane** is a two dimensional geometric object. It has infinite length and infinite width but no thickness.

Definitions:

- **Parallel lines** are lines that lie in the same plane but do not intersect. (Symbol: $\parallel$)
- **Perpendicular lines** are two lines that meet to form congruent adjacent angles. (Symbol: $\perp$)

Theorem: If two lines are perpendicular, then they meet to form _____ angles.

GIVEN: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E.
PROVE: $\angle AEC$ is a right angle

PROOF	
Statements	**Reasons**
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E.	**1.**
2. $\angle AEC \cong \angle CEB$	**2.**
3. $m\angle AEC = m\angle CEB$	**3.**
4. $\angle AEB$ is a straight angle and $m\angle AEB = 180^\circ$	**4.**
5. $m\angle AEC + m\angle CEB = m\angle AEB$	**5.**
6. $m\angle AEC + m\angle CEB = 180^\circ$	**6.**
7. $m\angle AEC + m\angle AEC = 180^\circ$ or $2 \cdot m\angle AEC = 180^\circ$	**7.**
8. $m\angle AEC = 90^\circ$	**8.**
9. $\angle AEC$ is a right angle	**9.**

A _____ "connects" two elements of a set of objects.

Relation R	***Objects Related***	***Example of Relationship***
is equal to	numbers	
is greater to	numbers	
is perpendicular to	lines	
is complementary to	angles	
is congruent to	line segments	
is a brother of	people	

There are three special properties that *may* exist for a given relation.

1. **Reflexive property:** $a\boldsymbol{R}a$

Example: $5 = 5$, equality of numbers has a reflexive property.

2. **Symmetric property:** If $a\boldsymbol{R}b$, then $b\boldsymbol{R}a$.

Example: If $n \perp m$, then $m \perp n$, perpendicular lines have the symmetric property).

3. **Transitive property:** If $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$.

Example: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$, congruence of angle is transitive.

Example: Does the relation "is a brother of " have
a. a reflexive property (consider one male)?
b. a symmetric property (consider two males)?
c. a transitive property (consider three males)?

a. $a\boldsymbol{R}a$

_____ is a brother of _____

b. If $a\boldsymbol{R}b$, then $b\boldsymbol{R}a$.

If _____ is a brother of _____, then

_____ is a brother of _____.

c. If $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$.

If _____ is a brother of _____ and

_____ is a brother of _____, then

_____ is a brother of _____.

Example: Does the relation "is complementary to" have
a. a reflexive property (consider one angle)?
b. a symmetric property (consider two angles)?
c. a transitive property (consider three angles)?

a. $a\boldsymbol{R}a$

_____ is complementary to _____

b. If $a\boldsymbol{R}b$, then $b\boldsymbol{R}a$.

If _____ is complementary to _____, then

_____ is complementary to _____.

c. If $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$.

If _____ is complementary to _____ and

_____ is complementary to _____, then

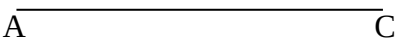
_____ is complementary to _____.

Definition: The **perpendicular bisector** of a segment is a line (or a segment) that is perpendicular to a given segment and divides it into two congruent segments.

Theorem: The perpendicular bisector of a line segment is unique.

Example:

a. How many bisectors does a segment have?



b. How many perpendicular bisectors does a segment have?

c. How many bisectors does a line have?