Math 1312
Section 1.6
Relationships: Perpendicular Lines

A **plane** is a two dimensional geometric object. It has infinite length and infinite width but no thickness.

$\ell \parallel m$ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

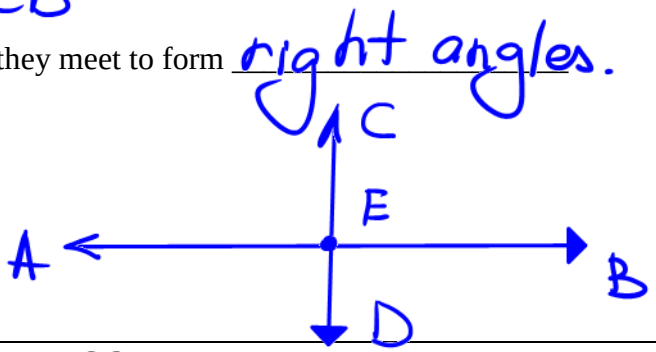
Definitions:

- **Parallel lines** are lines that lie in the same plane but do not intersect. (Symbol: $\parallel$)
- **Perpendicular lines** are two lines that meet to form congruent adjacent angles. (Symbol: $\perp$)

$\ell \perp m$ $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Theorem: If two lines are perpendicular, then they meet to form right angles. angles.

GIVEN: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E.
PROVE: $\angle AEC$ is a right angle



PROOF	
Statements	**Reasons**
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ intersecting at E.	**1.** Given
2. $\angle AEC \cong \angle CEB$	**2.** Def. of $\perp$ line
3. $m\angle AEC = m\angle CEB$	**3.** Def. of $\cong$ $\angle$s.
4. $\angle AEB$ is a straight angle and $m\angle AEB = 180^\circ$	**4.** A – E – B Def. of a str. $\angle$
5. $m\angle AEC + m\angle CEB = m\angle AEB$	**5.** Angle-Add. Post.
6. $m\angle AEC + m\angle CEB = 180^\circ$	**6.** Substitution
7. $m\angle AEC + m\angle AEC = 180^\circ$ or $2 \cdot m\angle AEC = 180^\circ$	**7.** Substitution
8. $m\angle AEC = 90^\circ$	**8.** Division Prop. of Eq.
9. $\angle AEC$ is a right angle	**9.** Def. of a right $\angle$.

A Relation "connects" two elements of a set of objects.

Relation R	*Objects Related*	*Example of Relationship*
is equal to	numbers	$6 = 6$
is greater than	numbers	$7 > 5$
is perpendicular to	lines	$\ell \perp m$
is complementary to	angles	$\angle 1$ is comp. to $\angle 2$
is congruent to	line segments	$\overline{AB} \cong \overline{CD}$
is a brother of	people	Mike is a brother John.

There are three special properties that *may* exist for a given relation.

1. **Reflexive property:** aRa

Example: $5 = 5$, equality of numbers has a reflexive property.

2. **Symmetric property:** If aRb, then bRa.

Example: If $n \perp m$, then $m \perp n$, perpendicular lines have the symmetric property).

3. **Transitive property:** If aRb and bRc, then aRc.

Example: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$, congruence of angle is transitive.

Example: Does the relation "is a brother of " have
a. a reflexive property (consider one male)? ✗
b. a symmetric property (consider two males)? ✓
c. a transitive property (consider three males)? ✓

a. aRa

Ike is a brother of Ike NO

b. If $a\boldsymbol{R}b$, then $b\boldsymbol{R}a$.

If Ike is a brother of Mike, then YES

Mike is a brother of Ike.

c. If $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$.

If Ike is a brother of Mike and

Mike is a brother of Dave, then YES

Ike is a brother of Dave.

Example: Does the relation "is complementary to" have
a. a reflexive property (consider one angle)? ✗
b. a symmetric property (consider two angles)? ✓
c. a transitive property (consider three angles)? ✗

a. $a\boldsymbol{R}a$

$\angle 1$ is complementary to $\angle 1$ NO

$m\angle 1 + m\angle 1 = 90°$

b. If $a\boldsymbol{R}b$, then $b\boldsymbol{R}a$.

If $\angle 1$ is complementary to $\angle 2$, then YES

$\angle 2$ is complementary to $\angle 1$.

If $m\angle 1 + m\angle 2 = 90°$, then

$m\angle 2 + m\angle 1 = 90°$

c. If $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$.

If $\angle 1$ is complementary to $\angle 2$ and

$\angle 2$ is complementary to $\angle 3$, then NO

$\angle 1$ is complementary to $\angle 3$.

$m\angle 1 (60) + m\angle 2 (30) = 90°$

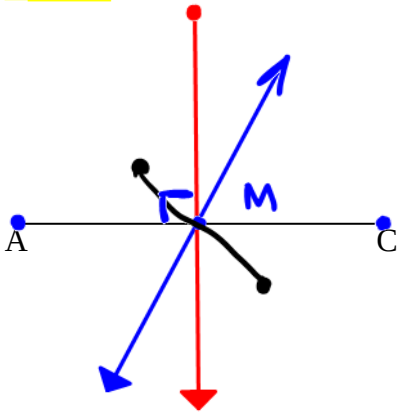
$m\angle 2 (30) + m\angle 3 (60) = 90°$

$m\angle 1 = m\angle 2$

$m\angle 1 + m\angle 3 = 90$

Definition: The **perpendicular bisector** of a segment is a line (or a segment) that is perpendicular to a given segment and divides it into two congruent segments.

Theorem: The perpendicular bisector of a line segment is unique.



Example:

a. How many bisectors does a segment have?

∞ many

b. How many perpendicular bisectors does a segment have?

one

c. How many bisectors does a line have?

none