Math 1312
Section 1.7
The Formal Proof of a Theorem

When a statement has the form "$If\ H, then\ C$," the **hypothesis** is H and the **conclusion** is C.

The hypothesis of a statement describes _____;

The conclusion describes what you need to _____.

Some theorems must be reworded into "$If\ \ldots, then\ \ldots$" form.

Examples: Give the hypothesis and conclusion for each statement.

a. If x and y are any two quantities with $x = y$, then x can be substituted for y in any expression containing y.

b. Vertical angles are congruent.

   Reworded:

c. Two lines with slopes m_1 and m_2 are parallel if $m_1 = m_2$.

   Reworded:

Recall: Conditional statements have a hypothesis (P) and a conclusion (Q) and are in the form:

$$If\ P, then\ Q.$$

We can write this with symbols: $P \rightarrow Q$.

Definition: The **converse** of a statement "$If\ P, then\ Q$" is "$If\ Q, then\ P$."

That is, the converse of the given statement interchanges the hypothesis and conclusion. The words "if" and "then" do not move.

Example:
Theorem 1.6.1: If two lines are perpendicular, then they meet to form right angles.

Theorem 1.7.1: If two lines meet to form right angles, then these lines are perpendicular.

Example: Write the converse of the statement:
If a person lives in Houston, then that person lives in Texas.

Theorem 1.7.2: If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.

Theorem 1.7.3: If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.

Theorem 1.7.4: Any two right angles are congruent.

Theorem 1.7.5: If the exterior sides of two acute adjacent angles form perpendicular rays, then these angles are complementary.

Theorem 1.7.6: If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.

Theorem 1.7.7: If two lines segments are congruent, then their midpoints separate these into four congruent segments.

Theorem 1.7.8: If two angles are congruent, then their bisectors separate these angles into four congruent angles.