Math 1312
Section 1.7
The Formal Proof of a Theorem

When a statement has the form "$If\ H, then\ C$," the **hypothesis** is H and the **conclusion** is C.

The hypothesis of a statement describes given;

The conclusion describes what you need to prove.

Some theorems must be reworded into "$If\ \dots, then\ \dots$" form.

Examples: Give the hypothesis and conclusion for each statement.

a. If x and y are any two quantities with $x = y$, then x can be substituted for y in any expression containing y.

b. Vertical angles are congruent.

Reworded:

If two $\angle$s are vertical, then they are $\cong$.

c. Two lines with slopes m_1 and m_2 are parallel if $m_1 = m_2$.

Reworded:

If $m_1 = m_2$, then two lines with slopes m_1 & m_2 are $\parallel$.

Recall: Conditional statements have a hypothesis (P) and a conclusion (Q) and are in the form:

$$If\ P, then\ Q.$$

We can write this with symbols: $P \rightarrow Q$.

Definition: The **converse** of a statement "$If\ P, then\ Q$" is "$If\ Q, then\ P$."

That is, the converse of the given statement interchanges the hypothesis and conclusion. The words "if" and "then" do not move.

Example:

Theorem 1.6.1: If two lines are perpendicular, then they meet to form right angles. True

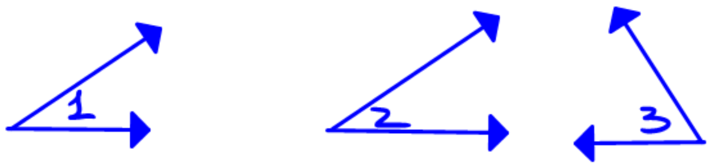
Theorem 1.7.1: If two lines meet to form right angles, then these lines are perpendicular. True

Example: Write the converse of the statement:

If a person lives in Houston, then that person lives in Texas. True

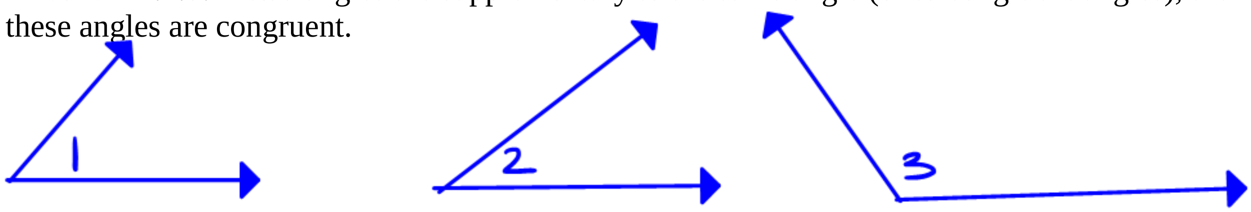
If a person lives in Texas, then that person lives in Houston. False

Theorem 1.7.2: If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.



$\angle 1$ is comp. to $\angle 3$
$\angle 2$ is comp. to $\angle 3$
$\therefore \angle 1 \cong \angle 2$

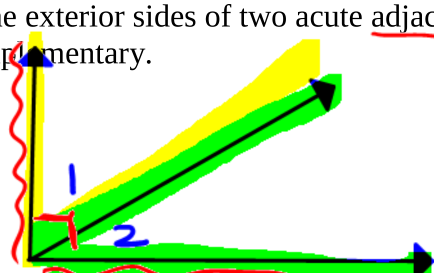
Theorem 1.7.3: If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.



$\angle 1$ is supp. to $\angle 3$
$\angle 2$ is supp. to $\angle 3$
$\angle 1 \cong \angle 2$

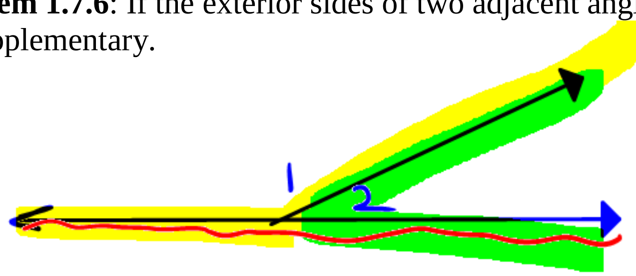
Theorem 1.7.4: Any two right angles are congruent.

Theorem 1.7.5: If the exterior sides of two acute adjacent angles form perpendicular rays, then these angles are complementary.



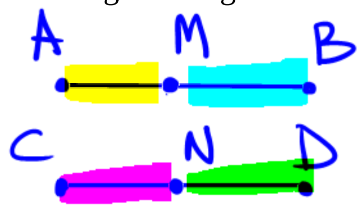
$m\angle 1 + m\angle 2 = 90$

Theorem 1.7.6: If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.



$m\angle 1 + m\angle 2 = 180$

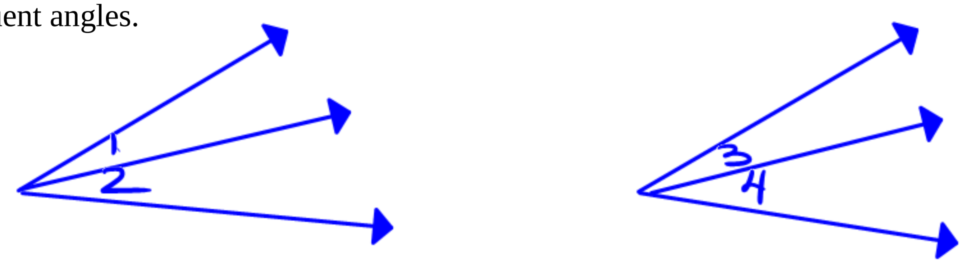
Theorem 1.7.7: If two lines segments are congruent, then their midpoints separate these into four congruent segments.



Given: $\overline{AB} \cong \overline{CD}$

Conc.: $AM = MB = CN = ND$

Theorem 1.7.8: If two angles are congruent, then their bisectors separate these angles into four congruent angles.



$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$