Math 1312
Section 2.2
Indirect Proof

Recall: $P \to Q$ represents the **conditional statement** "$If\ P, then\ Q$".
$\sim P$ represents the **negation** of P.

Definitions:

- The ______________________ of a conditional statement is formed by *negating* the hypothesis and *negating* the conclusion of the original statement.
  In other words, the word "not" is added to both parts of the sentence; the words "if" and "then" do **not** move.

- The ______________________ of a statement is formed by *interchanging* the hypothesis and conclusion.
  In other words converse switches the parts of the conditional statement; the words "if" and "then" do **not** move.

- The ______________________ of a conditional statement is formed by *negating* both the hypothesis and the conclusion, ***and*** then *interchanging* the resulting negations.
  In other words, the contrapositive negates and switches the parts of the sentence.

Conditional	$P \to Q$	If P, then Q.
Converse	$Q \to P$	If Q, then P.
Inverse	$\sim P \to \sim Q$	If not P, then not Q.
Contrapositive	$\sim Q \to \sim P$	If not Q, then not P.

Example: Give the inverse, converse and contrapositive for the following conditional statement. Then classify each as true or false.

If a polygon is a square, then it has four sides.

CONVERSE:

INVERSE:

CONTRAPOSITIVE:

$$\textit{If } x > 3, \textit{ then } x \neq 0.$$

CONVERSE:

INVERSE:

CONTRAPOSITIVE:

FACT: If a conditional statement is true, its contrapositive is TRUE!

The Law of Detachment:
1. $P \to Q$
2. P
Conclusion: $\therefore\ Q$

The Law of Negative Inference:
1. $P \to Q$
2. $\sim Q$
Conclusion: $\therefore\ \sim P$

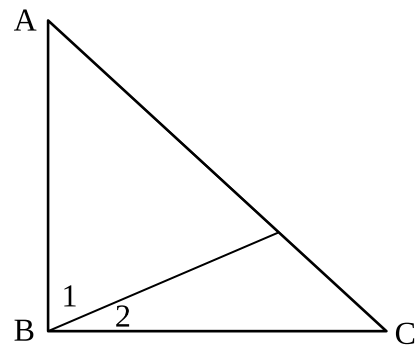
Example: Use the Law of Negative Inference to draw a conclusion.

1. If two angles are vertical angles, then they are congruent.
2. $\angle 1$ and $\angle 2$ are not congruent.

Conclusion:

Indirect Proofs use the law of negative inference.

Example: Complete a formal proof of the following statement.

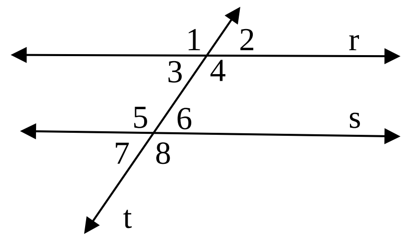


GIVEN: $\angle ABC$ is not a right angle.
PROVE: $\angle 1$ and $\angle 2$are not complementary.

PROOF:

Example: Complete a formal proof of the following statement.

GIVEN: $\angle 4 \ncong \angle 8$
PROVE: $r \nparallel s$



PROOF: