Math 1312
Section 2.2
Indirect Proof

Recall: $P \rightarrow Q$ represents the **conditional statement** "$If\ P, then\ Q$".
$\sim P$ represents the **negation** of P.

Definitions:

- The ___inverse___ of a conditional statement is formed by *negating* the hypothesis and *negating* the conclusion of the original statement.
  In other words, the word "not" is added to both parts of the sentence; the words "if" and "then" do **not** move.

- The ___converse___ of a statement is formed by *interchanging* the hypothesis and conclusion.
  In other words converse switches the parts of the conditional statement; the words "if" and "then" do **not** move.

- The ___contrapositive___ of a conditional statement is formed by *negating* both the hypothesis and the conclusion, ***and*** then *interchanging* the resulting negations.
  In other words, the contrapositive negates and switches the parts of the sentence.

Conditional	$P \rightarrow Q$	If P, then Q.
Converse	$Q \rightarrow P$	If Q, then P.
Inverse	$\sim P \rightarrow \sim Q$	If not P, then not Q.
Contrapositive	$\sim Q \rightarrow \sim P$	If not Q, then not P.

Example: Give the inverse, converse and contrapositive for the following conditional statement. Then classify each as true or false.

$P \rightarrow Q$

If $\underbrace{\text{a polygon is a square}}_{P}$, then $\underbrace{\text{it has four sides}}_{Q}$. True

CONVERSE:

$Q \rightarrow P$ If a polygon has 4 sides, then it is a square. False

INVERSE: If a polygon is not a square, then it does not have 4 sides. False

$\sim P \to \sim Q$

CONTRAPOSITIVE: If a polygon does not have 4 sides, then it is not a square. True

$\sim Q \to \sim P$

If x > 3, then x ≠ 0. True

CONVERSE: If $x \neq 0$, then $x > 3$. False

$Q \to P$

INVERSE: If $x \leq 3$, then $x = 0$. False

$\sim P \to \sim Q$

CONTRAPOSITIVE: If $x = 0$, then $x \leq 3$. True

$\sim Q \to \sim P$

FACT: If a conditional statement is true, its contrapositive is TRUE!

The Law of Detachment:
1. $P \to Q$
2. P

Conclusion: $\therefore\ Q$

The Law of Negative Inference:
1. $P \to Q$
2. $\sim Q$

Conclusion: $\therefore\ \sim P$

Example: Use the Law of Negative Inference to draw a conclusion.

P Q

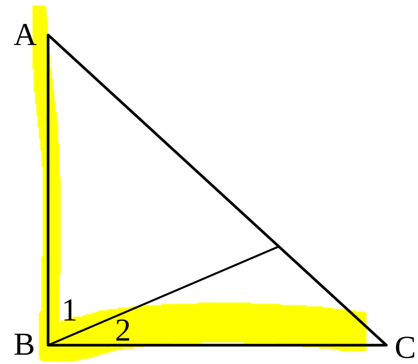
1. If two angles are vertical angles, then they are congruent.
2. ∠1 and ∠2 are not congruent. $\sim Q$

Conclusion: ∠1 & ∠2 are not vertical.

Indirect Proofs use the law of negative inference.

Example: Complete a formal proof of the following statement.

GIVEN: $\angle ABC$ is not a right angle.
PROVE: $\angle 1$ and $\angle 2$are not complementary.



PROOF:

Assume $\angle 1$ & $\angle 2$ are complementary.

$m\angle 1 + m\angle 2 = 90°$

$m\angle 1 + m\angle 2 = m\angle ABC$ (Angl.-Add. Post.)

$m\angle ABC = 90°$

$\angle ABC$ is a right $\angle$ BUT

$\angle ABC$ is not a right $\angle$.

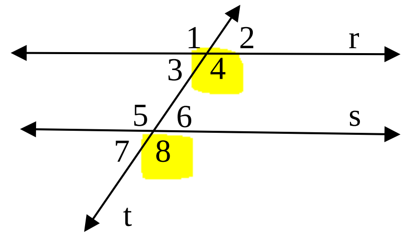
Hence the assumed statement, which claims that $\angle 1$ & $\angle 2$ are comp. is false.

It follows that

$\angle 1$ & $\angle 2$ are not complementary ∎.

Example: Complete a formal proof of the following statement.

GIVEN: $\angle 4 \ncong \angle 8$
PROVE: $r \nparallel s$



PROOF:

Assume $r \parallel s$.

Then it follows $\angle 4 \cong \angle 8$

BUT $\angle 4 \ncong \angle 8$.

Hence the assumed statement that claims that $r \parallel s$ is false and $r \nparallel s$. ■.