Math 1312
Section 2.3
Proving Lines Parallel

Suppose we wish to prove that two lines are parallel rather than establish an angle relationship as in section 2.1.
The theorems that allow that have the form "*If …, then these lines are parallel.*"

Theorem: If two lines are cut by a transversal so that the ***corresponding*** angles are ***congruent***, then these lines are parallel.

Theorem: If two lines are cut by a transversal so that the ***alternate interior*** angles are ***congruent***, then these lines are parallel.

Theorem: If two lines are cut by a transversal so that the ***alternate exterior*** angles are ***congruent***, then these lines are parallel.

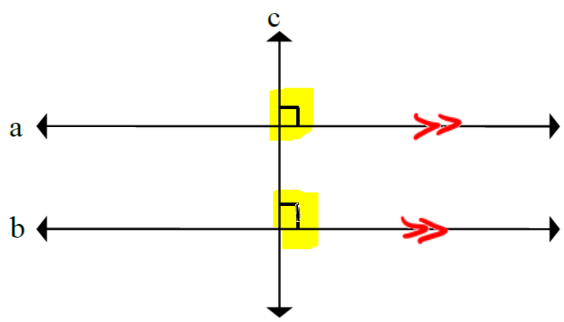
Theorem: If two lines are cut by a transversal so that the ***interior angles on one side of the transversal*** are ***supplementary***, then these lines are parallel.

Theorem: If two lines are cut by a transversal so that the ***exterior angles on one side of the transversal*** are ***supplementary***, then these lines are parallel.

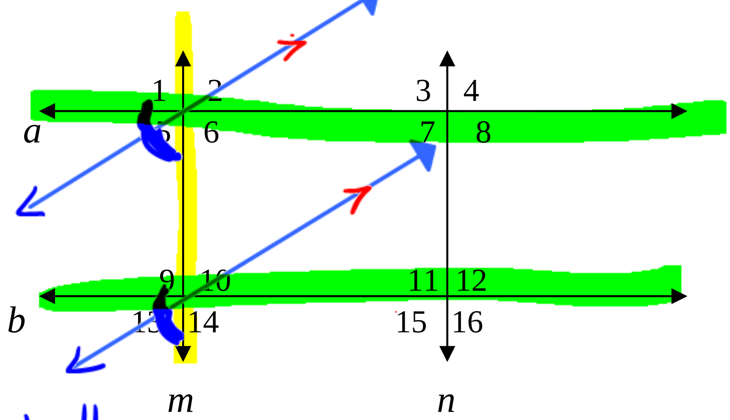
Theorem: If two coplanar lines are each ***perpendicular to a third line***, then there lines are parallel to each other.

GIVEN: $a \perp c$; $b \perp c$

PROVE: $a \parallel b$



Example: Name the lines (if any) that must be parallel under the given conditions.



a. $\angle 1 \cong \angle 3$ $m \parallel n$

corresponding

b. $\angle 4 \cong \angle 15$ $a \parallel b$

alt. extr.

c. $\angle 10 \cong \angle 13$ none

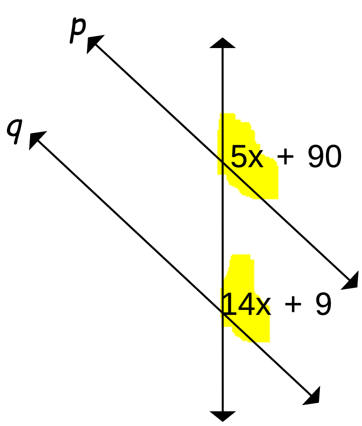
vertical

d. $a \perp m$ and $b \perp m$ $a \parallel b$

e. The bisectors of $\angle 5$ and $\angle 13$ are parallel.

$a \parallel b$

Example: Find the value of x and the measure of each angle that will make $p \parallel q$.



$5x + 90 = 14x + 9$

$90 = 9x + 9$

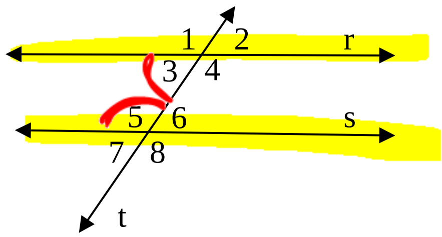
$81 = 9x$

$\underline{x = 9}$ $135°$

If $x = 9$, then $p \parallel q$.

If $x \neq 9$, then $p \nparallel q$.

Example: Determine the values of x and/or the angle(s) so that the line r will be parallel to s.



a. If $m\angle 1 = 107^\circ$, find $m\angle 5$. $= 107^\circ$

corresp.

b. If $m\angle 4 = 106^\circ$, find $m\angle 6$. $m\angle 6 = 180 - 106 = 74^\circ$

consecutive interior

c. If $m\angle 2 = 72^\circ$ and $m\angle 7 = (4x + 20)^\circ$.

alt. ext.

$m\angle 7 = m\angle 2$

$4x + 20 = 72$

$4x = 52$

$\boxed{x = 13}$ $\boxed{m\angle 7 = 72^\circ}$

d. If $m\angle 3 = (2x + 26)^\circ$ and $m\angle 5 = 6(x - 1)^\circ$

consecutive interior

$m\angle 3 + m\angle 5 = 180$

$2x + 26 + 6(x-1) = 180$

$2x + 26 + 6x - 6 = 180$

$8x + 20 = 180$

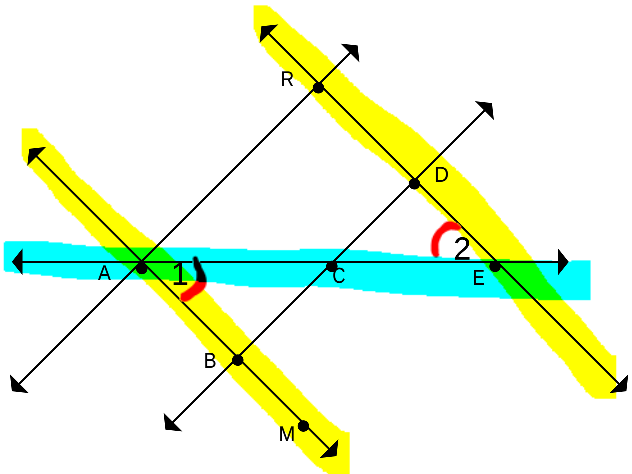
$8x = 160$

$\boxed{x = 20}$

$m\angle 3 = 2(20) + 26$
$= 66^\circ$

$m\angle 5 = 6(20-1)$
$= 114^\circ$

Example: If $\angle 1 \cong \angle 2$, then which lines must be parallel?

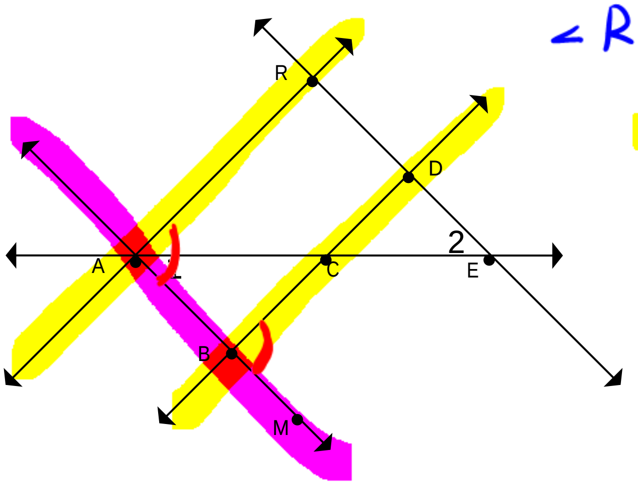


$\angle 1$ & $\angle 2$ are alt. int.

$\angle 1 \cong \angle 2$

$\overleftrightarrow{RE} \parallel \overleftrightarrow{AM}$

Example: If $\angle RAB \cong \angle CBM$, then which lines must be parallel?



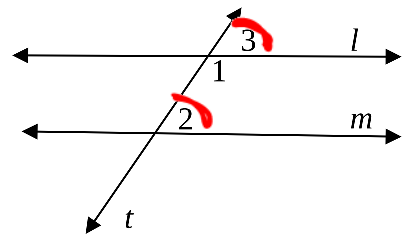
$\angle RAB$ & $\angle CBM$ are corresp.

$\angle RAB \cong \angle CBM$

$\overleftrightarrow{AR} \parallel \overleftrightarrow{DC}$

Theorem: If two lines are cut by a transversal so that the ~~*alternate*~~ consecutive ***interior*** angles are ~~***congruent***~~ supp., then these lines are parallel.

GIVEN: Lines l and m; transversal t
$\angle 1$ is supplementary to $\angle 2$
PROVE: $l \parallel m$



PROOF	
Statements	**Reasons**
1. l and m; trans. t; $\angle 1$ is supp. to $\angle 2$	**1.** Given
2. $\angle 1$ is supp. to $\angle 3$	**2.** If ext. sides of two adj $\angle$s form a straight line, then these $\angle$s are supp.
3. $\angle 2 \cong \angle 3$	**3.** If 2 $\angle$s are supp. to the same $\angle$, they are $\cong$.
4. $l \parallel m$	**4.** If 2 lines are cut by a trans. so that corresp. $\angle$s are $\cong$, then these lines are $\parallel$.