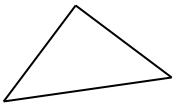
Math 1312
Section 2.4
The Angles of a Triangle

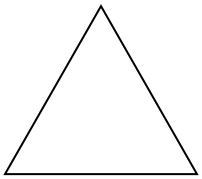
Definition: A triangle (symbol Δ) is the union of three line segments that are determined by three non-collinear points.

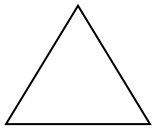
Each of the three noncollinear points is a_____ .
Line segments connecting any two points are the _____.

Classifying Triangles

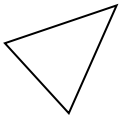
Triangles Classified by Congruent sides

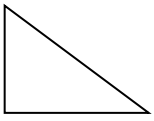
_____ No congruent sides 

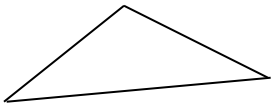
_____ Two congruent sides 

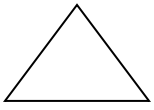
_____ Three congruent sides 

Triangles Classified by Angles

_____ all angles are acute 

_____ one right angle 

_____ one obtuse angle 

_____ all angles are congruent 

Question: Is equilateral triangle also an isosceles triangle?

A. Always true.
B. Sometimes true.
C. Never true

Question: Is isosceles triangle also an equilateral triangle?

A. Always true.
B. Sometimes true.
C. Never true

Theorem (Angle Sum Theorem): The sum of the measures of the interior angles of a triangle is 180°.

NOTE: we need an ***auxiliary*** (or helping) line drawn to prove this theorem.

Definition: When there is exactly one figure with certain properties, than such a figure is ***determined***.

Example: A line or a segment through a point NOT on a given line, parallel to a given line.

Explanation: Such a line is ***unique***, i.e. there is exactly one line with such property.

Definition: A figure is ***underdetermined*** if more than one possible figure is described.

Example: Draw a line that goes through the midpoint of a segment.

Explanation: You may draw infinitely many lines that contain a certain point (midpoint in the example).

Definition: A figure is ***overdetermined*** when it is not possible to draw a figure that satisfies all conditions stated.

Example: Draw a line that contains three noncolliner points.

Explanation: This is NOT possible. Noncolliner points are points NOT on the same line.

Definition: A theorem that follows directly from a previous theorem is a _____ of that theorem.

Corollary: Each angle in an equiangular triangle measures 60°.

Corollary: The acute angles in a right triangle are complementary.

Example: Classify the following triangles.

1. In ΔABC, $AB = 5$, $BC = 5$, and $m\angle B = 90^{\circ}$.

   _____ _____

2. In ΔPQR, $PQ = 5$, $QR = 6$, and $PR = 7$.

   _____ _____

3. In ΔDOG, $m\angle D = 49^{\circ}$ and $m\angle G = 51^{\circ}$.

   _____ _____

4. In ΔCAT, $m\angle A = 2x^{\circ}$, $m\angle C = (x - 23)^{\circ}$, and $m\angle A = (x - 17)^{\circ}$

   _____ _____

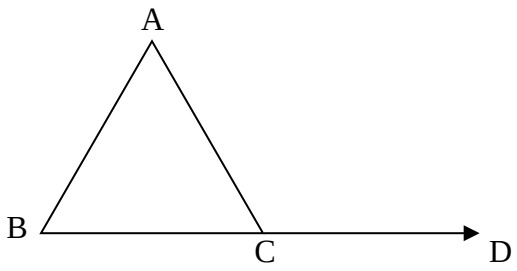
Corollary (Third Angle Theorem): If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Definition: An ***exterior angle*** is formed by one side of a triangle and the extension of another side.
Remote interior angles are the interior angles of a triangle that are not adjacent to a given angle.

Corollary: The measure of an exterior angle of a triangle equals the sum of the two measures of the two non-adjacent interior angles.

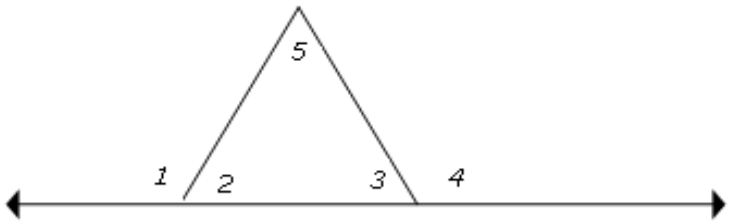
Example:

a. Name one exterior angle for triangle ABC.



b. Find x if $m\angle ACD = x^2 + 2x$, $m\angle A = x^2 - 2x$, and $m\angle B = 3x + 10$.

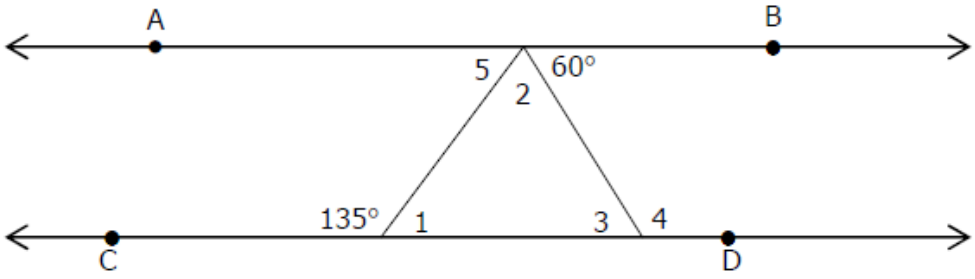
Example:
GIVEN: $m\angle 1 = 8(x + 2),\ m\angle 3 = 5x - 3, m\angle 5 = 5(x + 1) - 2.$
FIND: *x* and measures of angles 1, 2, 3, 4 and 5.



$m\angle 1 =$ _____ $m\angle 3 =$ _____

$m\angle 2 =$ _____ $m\angle 5 =$ _____

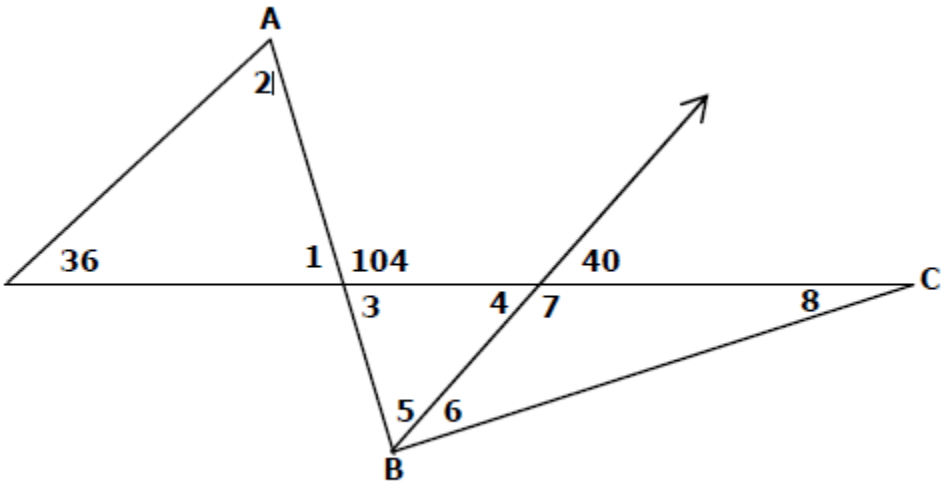
Example: Find the measure of each of the angles below, if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.



$m\angle 1 =$ _____ $m\angle 3 =$ _____

$m\angle 2 =$ _____ $m\angle 5 =$ _____

Example: If $\overline{AB} \perp \overline{BC}$, then find the measure of each angle in the figure below.



$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m\angle 3 =$ _____

$m\angle 5 =$ _____

$m\angle 6 =$ _____

$m\angle 7 =$ _____

$m\angle 8 =$ _____