## Math 1312 Section 2.5 Convex Polygons

**Definition:** A polygon is closed plane figure whose sides are line segments that intersect only at the endpoints.

A *convex polygon* has two properties:

- a) Every angle measures between  $0^{\circ}$  and  $180^{\circ}$ .
- b) A line segment joining two points of a convex polygon remains inside or on the boundary of the polygon.

## A concave polygon:

- a) Always has at least one reflex (between 180° and 360°) angle.
- b) A line segment joining two points of a concave polygon can contain points in the exterior of the polygon.

Example: Which figure is a convex polygon, concave polygon, or not a polygon?



Special names for polygons with fixed numbers of sides

Number of sides	Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
21	21-gon



**Theorem:** The total number of diagonals *D* in a polygon of n sides is given by the formula

$$D=\frac{n(n-3)}{2}.$$

**Example:** Find the number of diagonals for any decagon.

$$n. (h = 10)$$

$$D = \frac{10(10-3)}{2} = \frac{10(7)}{2} = 35$$

**Theorem:** The sum *S* of the measures of the interior angles of a polygon with *n* sides is given by

$$S = (n-2) \cdot 180^{\circ}$$

**Example:** Find the sum of the interior angles of the given polygon.

given - n find -S

given-S

a. triangle h=3

b. 11-gon.

S = (3-2) · 180° = 180°



Example: Find the number of sides a polygon has given the sum of the interior angles is 1980°.

 $\frac{1980}{180} = (n-2) \cdot \frac{180}{180}$   $\frac{180}{11} = n-2$  n = 13

S = 1980

## **Regular Polygons**

**Definition:** A *regular* polygon is a polygon that is both *equilateral* and *equiangular*.

**Example:** Which of the following polygons is regular?



**Corollary**: the measure I of each interior angle of a regular polygon or equiangular polygon of n sides

$$I = \frac{(n-2) \cdot 180^{\circ}}{n}.$$

**Example**: Find the measure of each of the interior angles of a stop sign.



**Example:** Each interior angle of a regular polygon is 150°. Find the number of sides.

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**Corollary**: The sum of the measured of the exterior angles of a polygon is 360°.

**Corollary:** The measure E of each exterior angle of a regular polygon of n sides is

$$E=\frac{360^{\circ}}{n}.$$

**Example:** Find the measure of each exterior angle of a regular nonagon.

$$E = \frac{360}{9} = 40^{\circ}$$



**Example:** If an interior angle of a regular polygon measures 165°, find the measure of an exterior angle and the number of sides.

165 =	_ (	(h -	2	).	180
				n	

$$E = 180 - 165 = 15^{\circ}$$
$$15 = \frac{360}{n}$$
$$h = \frac{360}{15} = 24$$