Math 1312
Section 3.1
Congruent Triangles

Definition:

If the six parts of one triangle are congruent to the corresponding six parts of another triangle, then the triangles are **congruent triangles**.

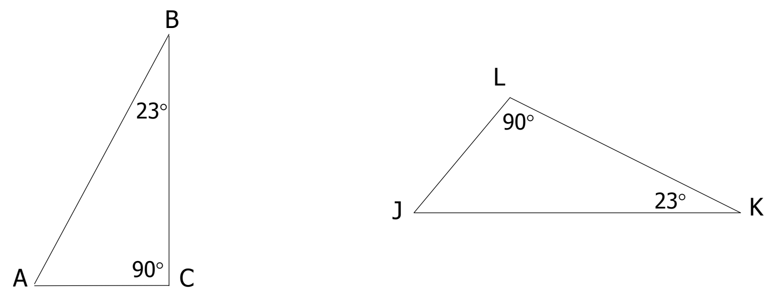
In other words:
- Congruent triangles are triangles that have the same size and the same shape.
- They are exact duplicates of each other.
- Such triangles can be moved on top of one another so that their corresponding parts line up exactly.

Definition:

To have a **correspondence** between two triangles, you must "match up" the angles and sides of one triangle with the angles and sides of the other triangle. Each corresponding angle and side must have the same measure.

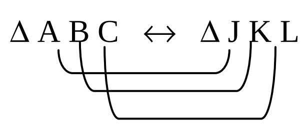
The order in which the letters are written matters since it shows which angles and sides of one triangle match up with the angles and sides of the other triangle.

Example 1:



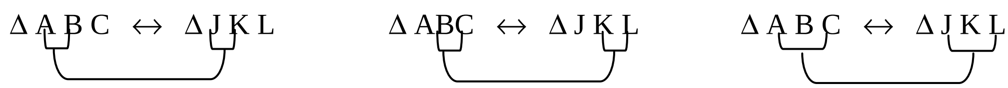
The correspondence between the above two triangles can be stated as $\Delta ABC \leftrightarrow \Delta JKL$.

If $\Delta ABC \leftrightarrow \Delta JKL$, the corresponding angles are:



$\angle A \leftrightarrow \angle J$, $\angle B \leftrightarrow \angle K$, $\angle C \leftrightarrow \angle L$

and the corresponding segments are:

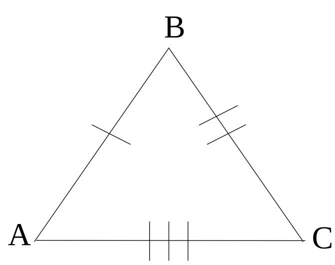


$\overline{AB} \leftrightarrow \overline{JK}$, $\overline{BC} \leftrightarrow \overline{KL}$, $\overline{AC} \leftrightarrow \overline{JL}$

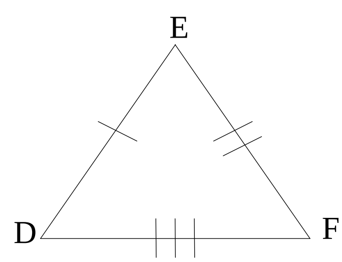
The correspondence may be written in more than one way: ΔCAB ↔ ΔLJK is the same as ΔABC ↔ ΔJKL.

Example 2:

ΔABC ≅ ΔDEF



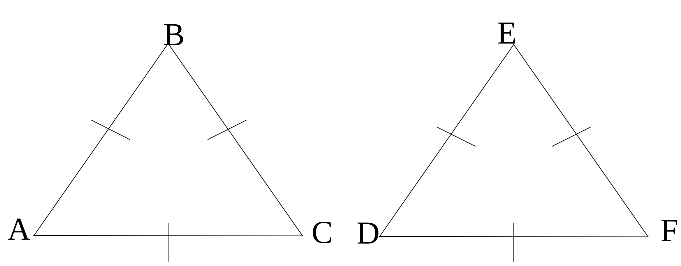
∠A ≅ ∠D   AB ≅ DE
∠B ≅ ∠E   BC ≅ EF
∠C ≅ ∠F   CA ≅ FD



Principle 1: (CPCTC) Corresponding parts of congruent triangles are congruent

Principle 2: (Side-Side-Side, SSS) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.

Example 3:

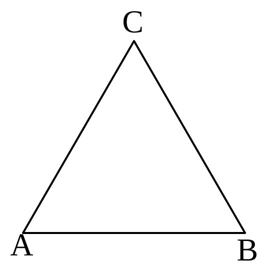


Since all three sides in ΔABC are congruent to all three sides in ΔDEF, then ΔABD ≅ ΔDEF

Definition:

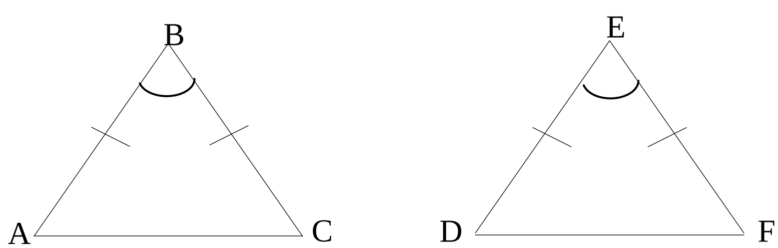
The angle made by two sides with a common vertex is the included angle.

Example 4:



Principle 3: (Side-Angle-Side, SAS) If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

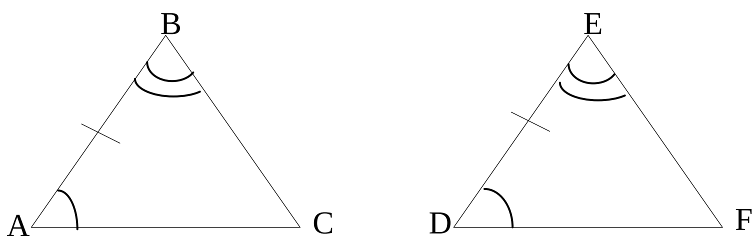
Example 5:



Since $\overline{AB} \cong \overline{DE}$, and
$\angle B \cong \angle E$, and
$\overline{BC} \cong \overline{EF}$, then
$\Delta ABC \cong \Delta DEF$

Principle 4: (Angle-Side-Angle, ASA) If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of a second triangle, the two triangles are congruent.

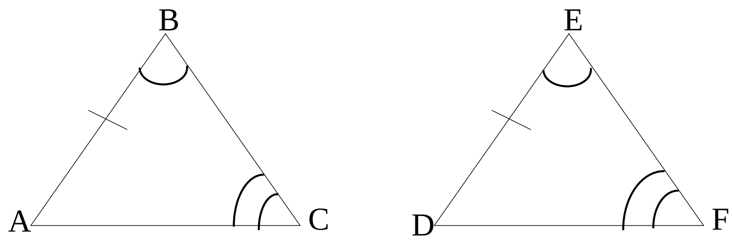
Example 6:



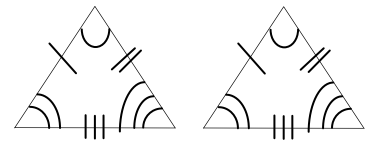
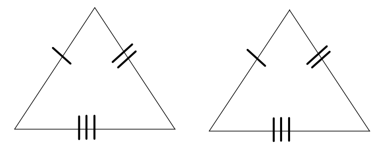
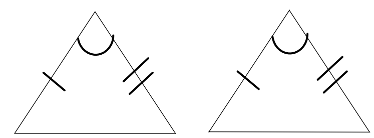
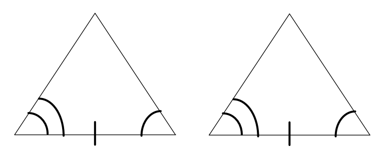
Since $\angle A \cong \angle D$, and
$\overline{AB} \cong \overline{DE}$, and
$\angle B \cong \angle E$, then
$\Delta ABC \cong \Delta DEF$

Principle 5: (Angle-Angle-Side, AAS) If two angles and a non-included side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

Example 6:



Since $\overline{AB} \cong \overline{DE}$, and
$\angle B \cong \angle E$, and
$\angle C \cong \angle F$, then
$\Delta ABC \cong \Delta DEF$

METHOD	QUALIFICATIONS	PICTURE
Def. of ≅ Δ	All six parts of one triangle must be congruent with all six parts of the other triangle.	
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.	
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.	
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.	
AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and non-included side of the other triangle