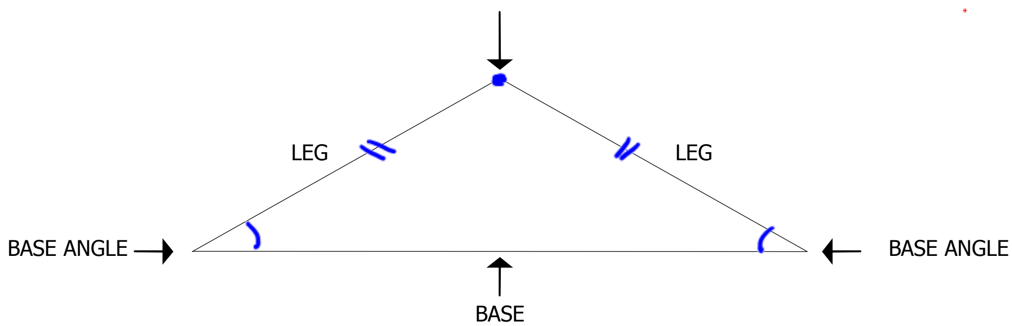
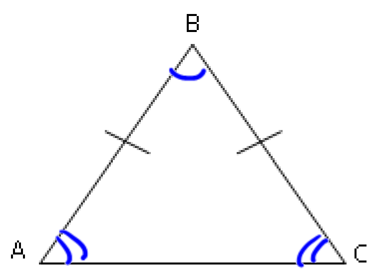
Math 1312
Section 3.3
Analyzing Isosceles Triangles

Definitions:

An **isosceles triangle** is a triangle having at least two congruent (of equal length) sides.
The two sides are called the **legs** and the third side is called the **base**.
The point at which the legs meet is the **vertex** and the angle there is the **vertex angle**.
The two angles that include the base are called the **base angles**.



Example: Name the parts of this isosceles triangle:



Legs: $\overline{AB}$, $\overline{BC}$
Base: $\overline{AC}$
Vertex: B

Vertex $\angle$: $\angle B$
Base $\angle$s: $\angle A$ & $\angle C$

Other important triangle parts:

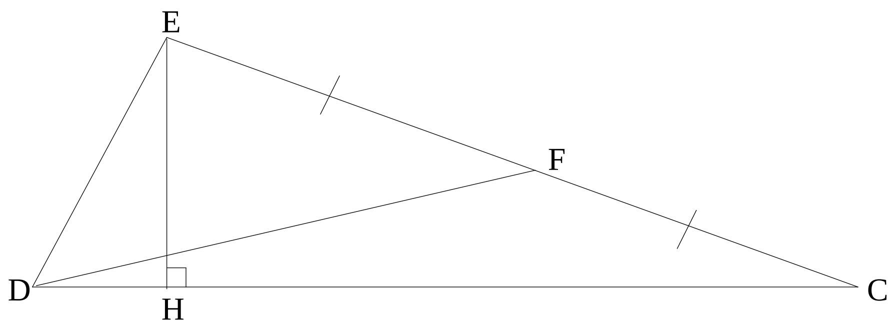
<u>Definitions:</u>

- **Median** is a segment that starts from an angle and goes to the <u>midpoint</u> of the opposite side.
- **Altitude** is a segment that starts from an angle and is <u>perpendicular to the opposite side</u>.
- **Angle bisector** of a triangle is a segment that <u>bisects an angle and goes to the opposite side</u>.
- **Perpendicular bisector** is a segment that <u>passes through the midpoint</u> of a side **AND** is <u>perpendicular to that side</u>.

Example: *Fill in the blanks.*

a) $\overline{DF}$ is median of ΔDEC.

b) $\overline{EH}$ is altitude of ΔDEC.



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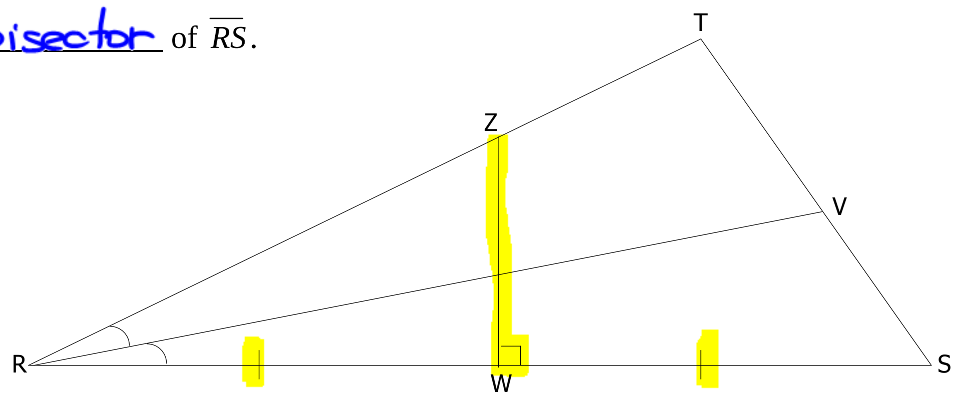
c) $\overline{RV}$ is angle bisector of ΔRST.

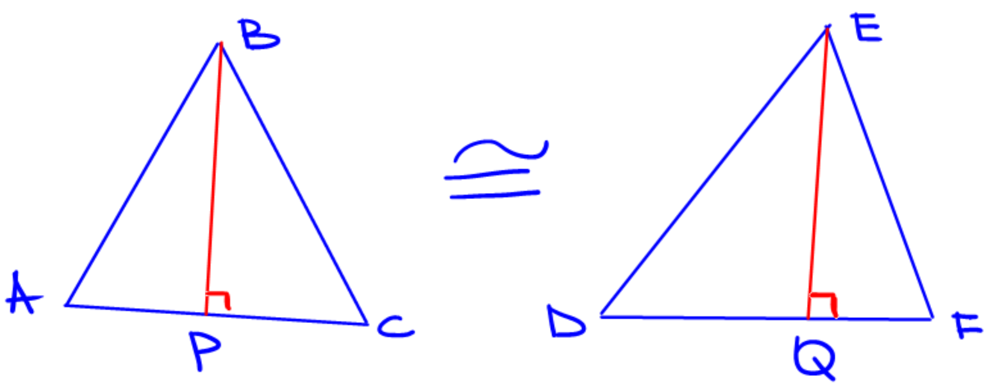
d) $\overline{WZ}$ is $\perp$ bisector of $\overline{RS}$.



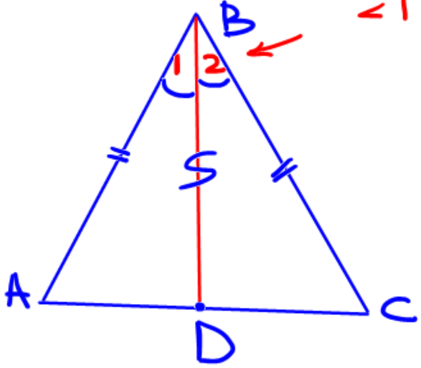
CPCTC

Theorem: Corresponding altitudes of congruent triangles are congruent.

$\overline{BP} \cong \overline{EQ}$



Theorem: The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.



$\angle 1 \cong \angle 2$

$\triangle ABD \cong \triangle CBD$

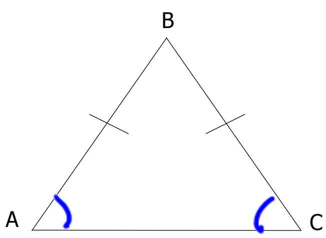
$\overline{AB} \cong \overline{CB}$

$\angle 1 \cong \angle 2$

$\overline{BD} \cong \overline{BD}$ (Identity)

by SAS

Isosceles Triangle Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

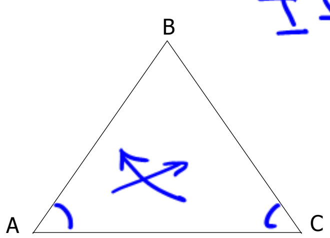


Given : $\triangle ABC$ with $\overline{AB} \cong \overline{BC}$

$(AB = BC)$

Then : $\angle A \cong \angle C$

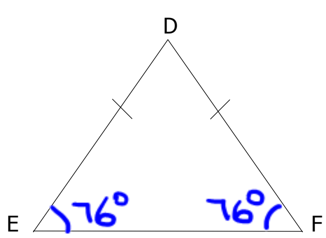
AND (converse): If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{BC}$.

Example:

ΔDEF is isosceles. $\angle D$ is the vertex angle. $m\angle E = 2x + 40$ and $m\angle F = 3x + 22$. Find the measures of each angle.



$m\angle E = m\angle F$

$2x + 40 = 3x + 22$

$40 = x + 22$

$18 = x$

$x = 18$

$m\angle E = 2(18) + 40$

$= 76°$

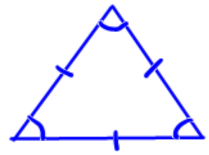
$m\angle F = 76°$

$m\angle D = 180 - 2(76)$

$= 28°$

Note:

- ➢ A triangle is equilateral **if and only if** it is equiangular.
- ➢ Each angle of an equilateral triangle measures 60°.

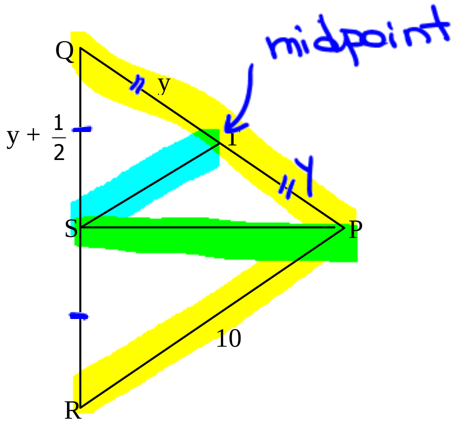


$$\frac{180^\circ}{3} = 60^\circ$$

Definition: The perimeter of a triangle is the sum of the lengths of all of its sides.

Example: In the figure below, $\overline{PQ} \cong \overline{PR}$, and $\overline{PS}$ and $\overline{ST}$ are medians. Find QT and QR.

y



$QP = 2y$

$QP = PR = 10$

$2y = 10$

$y = 5$

$\boxed{QT = 5}$

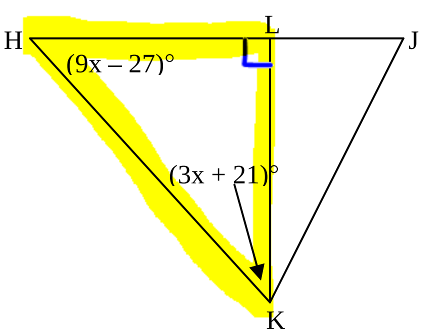
$QR = 2(QS)$

$= 2\left(y + \frac{1}{2}\right)$

$= 2y + 1$

$QR = 2(5) + 1 = \boxed{11}$

Example: $\overline{KL}$ is an altitude of ΔHJK. Find x.



ΔHLK is a right Δ.

$m\angle H + m\angle HKL = 90^\circ$

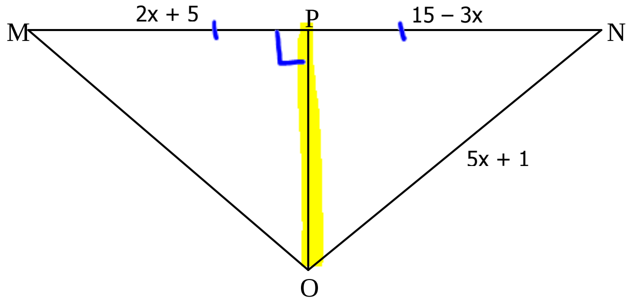
$9x - 27 + 3x + 21 = 90$

$12x - 6 = 90$

$12x = 96$

$\boxed{x = 8}$

Example: $\overline{PO}$ is the perpendicular bisector of $\overline{MN}$. Find x.



$MP = NP$

$2x+5 = 15-3x$

$5x+5 = 15$

$5x = 10$

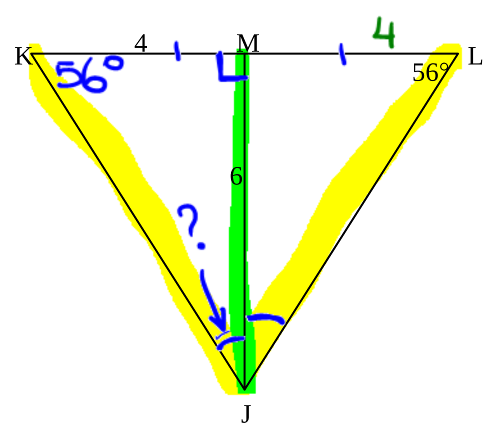
$\boxed{x = 2}$

Example: In ΔJKL, $\overline{JK} \cong \overline{JL}$, and $\overline{JM}$ is both a median, and altitude, and an angle bisector. Find the following.

a) $m\angle KMJ$ $= 90°$

b) KL $= 8$

c) $m\angle KJM$ $= 90 - 56 = 34°$



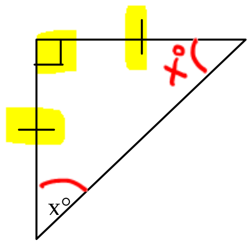
d) $m\angle KJL$ $= 2(34) = 68°$

e) $m\angle K$ $= 56°$

Example:

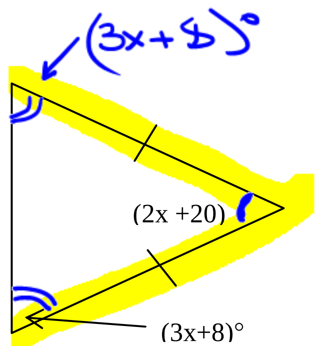
a) x = <u>45</u>

$$2x = 90$$
$$x = 45$$



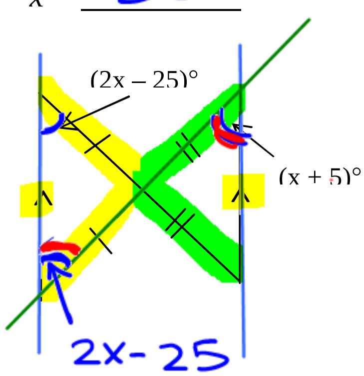
b) x = <u>18</u>

$$2(3x+8) + 2x+20 = 180$$
$$6x+16+2x+20 = 180$$
$$8x+36 = 180$$
$$8x = 144$$
$$x = 18$$



c) x = <u>30</u>

$$2x-25 = x+5$$
$$x-25 = 5$$
$$x = 30$$



Example: Use the figure below to find the angle measures if m∠1 = 30.°

m∠2 = $180 - 2(30) = 120°$

m∠3 = 30°

m∠4 = 60°

m∠5 = 60°

m∠6 = 60°

m∠7 = 120°

m∠8 = 90°