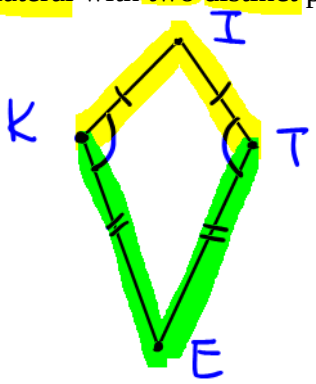
Math 1312
Section 4.2
The Parallelogram and Kite

Definition:
A kite is a quadrilateral with two distinct pairs of congruent adjacent sides.

Example 1:



$\overline{KI} \cong \overline{IT}$

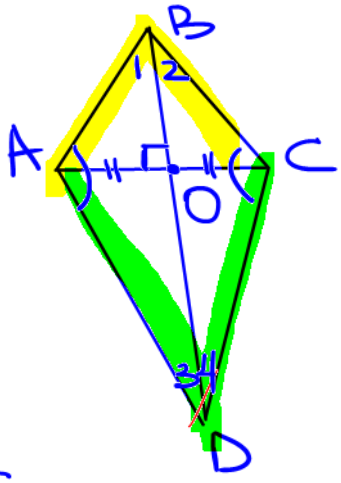
$\overline{KE} \cong \overline{TE}$

$\angle K \cong \angle T$

Theorem: In a kite, **one** pair of opposite angles are congruent.

Properties of Kites

1. NOT a parallelogram!



2. Two pairs of consecutive sides are congruent.

$AB = BC$ & $AD = CD$

3. The diagonals are perpendicular.

$\overline{BD} \perp \overline{AC}$

4. One diagonal is the perpendicular bisector of the other.

$\overline{BD}$ cuts $\overline{AC}$ in half, means $AO = OC$

$BO \neq OD$

5. One pair of opposite angles are congruent.

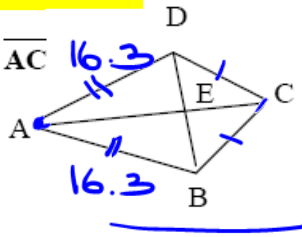
$\angle A \cong \angle C$

6. One of the diagonals bisects a pair of opposite angles.

$\angle 1 \cong \angle 2$ & $\angle 3 \cong \angle 4$

Example 2: Given a kite ABCD, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$:

a. If $\angle B = 90°$ and AB =8 and BC=6. Find the length of $\overline{AC}$



$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 6^2 = 64 + 36 = 100$

$AC^2 = 100$

$AC = 10$

(6, 8, 10)

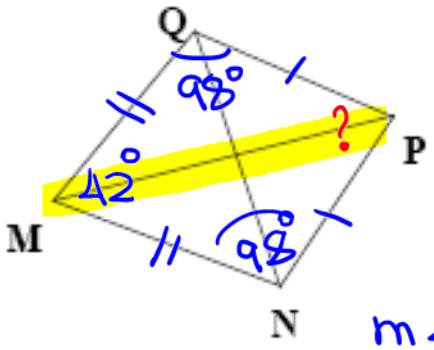
b. If AB = 16.3 and the perimeter of the kite is 54.7, find the lengths of DC, BC and AD.

$AB = AD = 16.3$

$BC = DC = \frac{54.7 - 2(16.3)}{2} = \frac{22.1}{2} = 11.05$

$BC = DC = 11.05$

Example: In kite MNPQ, $\overline{MP}$ is the perpendicular bisector of $\overline{NQ}$. If $m\angle QMN = 42°$ and $m\angle MNP = 98°$, find $m\angle NPQ$.



$S = (n-2)\cdot 180 \quad n = 4$

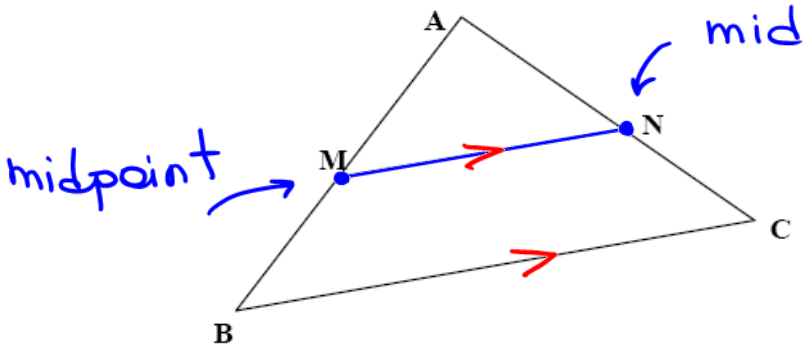
$S = (4-2)\cdot 180$

$= 2(180) = 360°$

$m\angle P = 360 - 42 - 2(98)$

$\boxed{m\angle P = 122°}$

Theorem 4.2.5: The segment that joins the midpoints of the two sides of a triangle is parallel to the third side and has a length equal to one half the length of the third side.



1. $\overline{MN} \parallel \overline{BC}$

2. $MN = \frac{1}{2}BC$

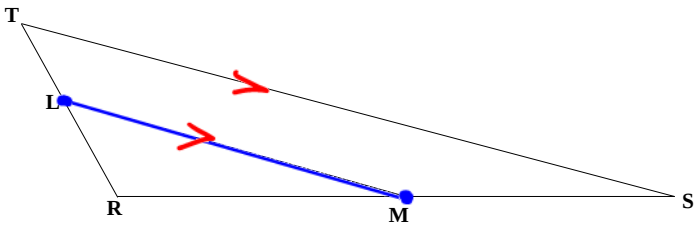
$BC = 2MN$

Popper 08

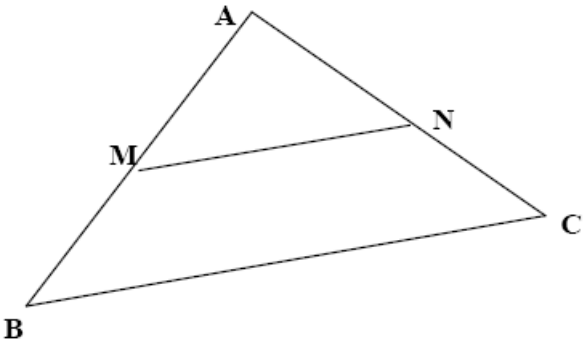
AB CDE AB CD

Clarification: in ΔTRS "M" is the midpoint of $\overline{RS}$ and "L" is the midpoint $\overline{RT}$.

By the above "rule", $\overline{ML}$ ll $\overline{ST}$ and ML = ½ ST. This can also be expressed as 2ML = ST.



Example 3. M and N are the midpoints of the sides $\overline{AB}$ and $\overline{BC}$ of ΔABC

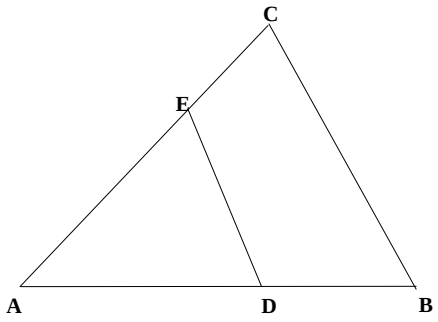


a. If MN = 7.3, find the length of $\overline{BC}$.

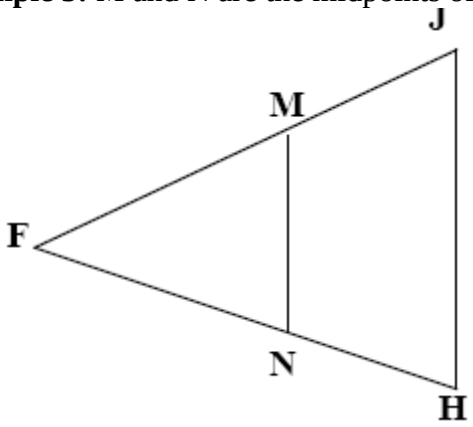
b. If BC = 4x + 6 and MN = x + 9, find the length of $\overline{BC}$.

Example 4:

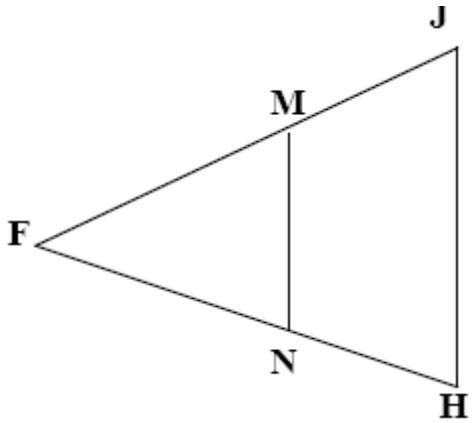
In the figure below, AE=8, CE=x, DA=6, and BA=12. Is ED ll CB?



Example 5: M and N are the midpoints of $\overline{FJ}$ and $\overline{FH}$

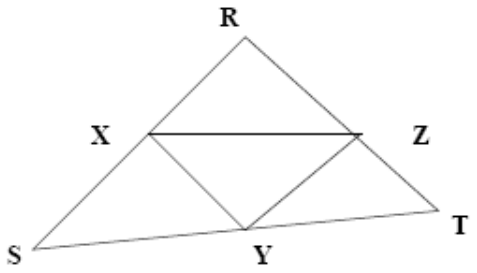


a. Given that ΔFHJ is isosceles, with $\overline{FJ} \cong \overline{FH}$, FM = 2y +3, NH = 5y−9 and JH = 2y. Find the perimeter of ΔFHJ .



b. Given JH = 12, $m\angle J = 80°$ and $m\angle F = 60°$. Find MN, $m\angle FMN$ and $m\angle FNM$.

Example 6: Use the following figure for both parts a and b. In ΔRST, X, Y and Z are the midpoints of the sides as shown.



a. If RS = 18 , RT =24, and ST = 26. Find XY, YZ, XZ and the perimeter of ΔXYZ.

b. If XY = 7.2 , XZ = 6.9, YZ = 5.1. Find RS, RT, ST and perimeter ΔRST.

Example 7:

In ΔABC, D is the midpoint of AB, E is the midpoint of BC, and F is the midpoint of AC. Find the perimeter of ΔDEF if AB = 24, BC = 32, and AC = 26.

Perimeter of ΔDEF = _____