

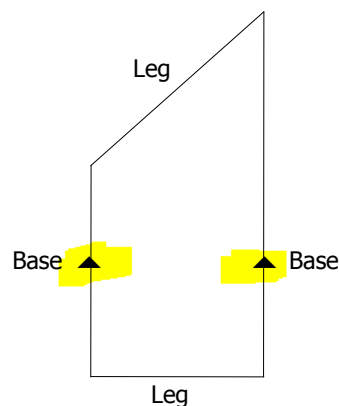
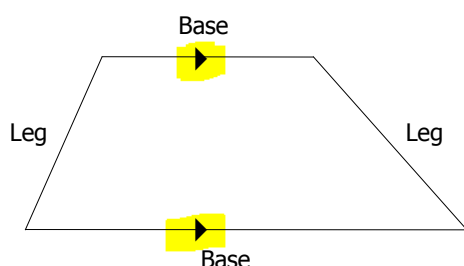
Math 1312
Section 4.4
The Trapezoid

Definition:

A **trapezoid** is a quadrilateral with **exactly one pair** of parallel sides.

The parallel sides are called **bases** of the trapezoid. The nonparallel sides are referred to as the **legs** of the trapezoid.

Example 1:

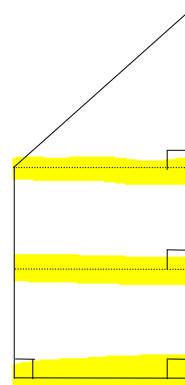
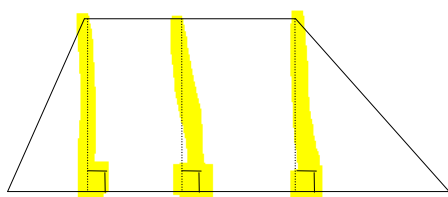


Definition:

An **altitude** of a trapezoid is a segment drawn from **any** point on one of the parallel sides (base) **perpendicular** to the opposite side (the other base).

An **infinite** number of **altitudes** may be drawn in a trapezoid.

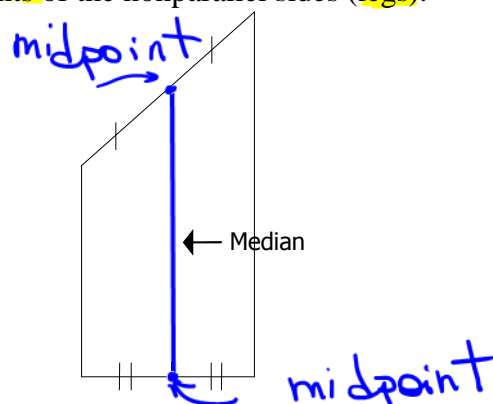
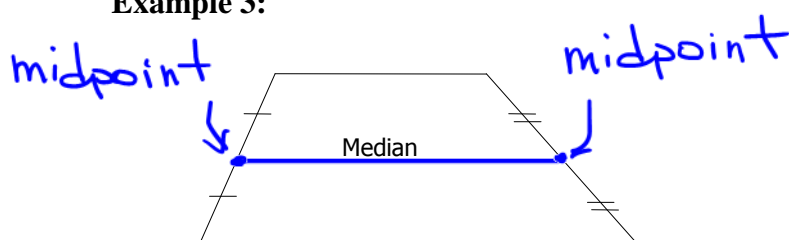
Example 2:



Definition:

A **median** of a trapezoid is the segment that joins the **midpoints** of the nonparallel sides (**legs**).

Example 3:



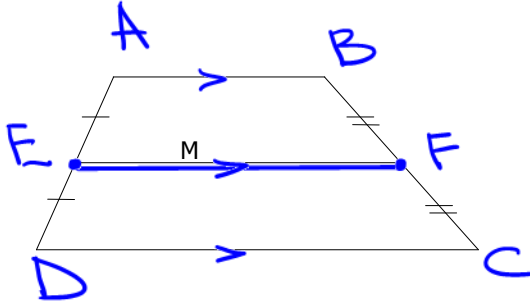
Theorem:



1. $\overline{MN} \parallel \overline{AC}$
2. $MN = \frac{1}{2} AC$

The median of a trapezoid is **parallel** to each base and the **length** of the median equals **one-half the sum** of the lengths of the **two bases**.

Example 4:



1. $\overline{EF} \parallel \overline{AB} \parallel \overline{DC}$
2. $EF = \frac{1}{2} (AB + DC)$

Definition:

An **isosceles** trapezoid is a trapezoid in which the **legs** (nonparallel sides) are **congruent**.

An isosceles trapezoid features some special properties not found in all trapezoids.

Properties of **Isosceles** Trapezoid

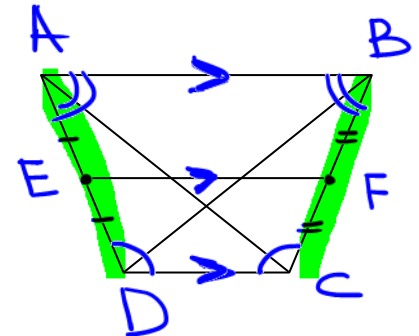
- 1. The legs are congruent.

$$AD = BC$$



2. The bases are parallel.

$$\overline{AB} \parallel \overline{DC}$$



- 3. The lower base angles of an isosceles trapezoid are congruent.

$$\angle ADC \cong \angle BCD$$

- 4. The upper base angles of an isosceles trapezoid are congruent.

$$\angle BAD \cong \angle ABC$$

- 5. The lower base angle is supplementary to **any** upper base angle.

$$m\angle ADC + m\angle BAD = 180^\circ \text{ etc}$$

- 6. The diagonals of an isosceles trapezoid are congruent.

$$AC = BD$$



7. The median is parallel to the bases

$$\overline{EF} \parallel \overline{AB} \parallel \overline{DC}$$

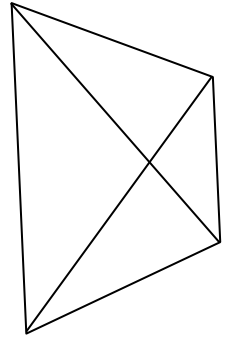


8. The length of the median equals one-half the sum of the lengths of the two bases.

$$EF = \frac{1}{2} (AB + DC)$$

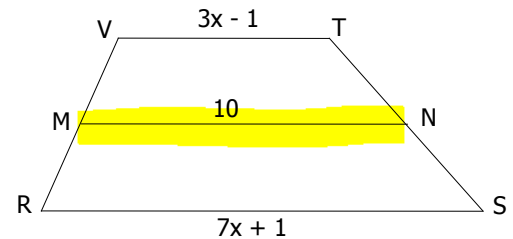
Proving that Trapezoid is isosceles

1. If **legs** of a trapezoid are **congruent** then it is an isosceles trapezoid.
with the same base
2. If **two base angles** of a trapezoid are **congruent**, then it is an isosceles trapezoid.
3. If the **diagonals** of a trapezoid are **congruent**, then it is an isosceles trapezoid.



Example 5:

Given trapezoid RSTV with median MN, find the value of "x".



$$MN = \frac{1}{2}(VT + RS)$$

$$2 \cdot 10 = \frac{1}{2}(3x + 7x + 1)$$

$$20 = 10x \quad \boxed{x = 2}$$

Example 6:

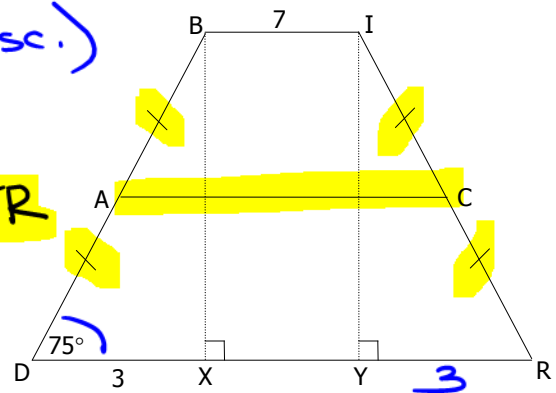
Find the missing measures of the given trapezoid.

(Isosc.)

a. $m\angle IRD = 75^\circ$

b. $YR = 3$

Note: $\triangle BXD \cong \triangle IYR$
by HL
by CPCTC



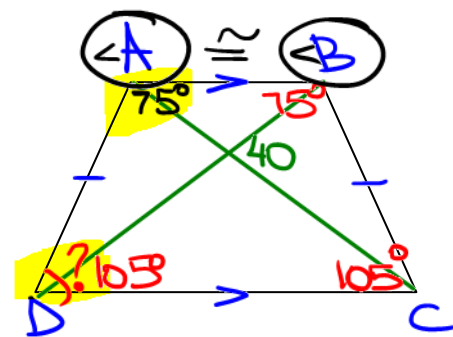
c. $DR = 3 + 7 + 3 = 13$

d. AC

$$= \frac{1}{2}(BI + DR) = \frac{1}{2}(7 + 13) = \frac{1}{2}(20) = 10$$

Example 7: Given an isosceles trapezoid $ABCD$ with legs \overline{AD} and \overline{BC} .

a. $m\angle DAB = 75^\circ$. Find the $m\angle ADC$. $= 180 - 75$
 $= 105^\circ$



b. $AC = 40$. Find BD . $= 40$

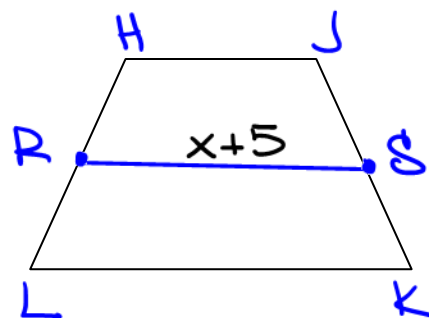
c. If $m\angle A = 6x + 25$ and $m\angle B = 8x + 15$. Find the measures of angles C and D .

$$\begin{array}{l|l} 6x + 25 = 8x + 15 & x = 5 \\ 25 = 2x + 15 & m\angle A = m\angle B = 6(5) + 25 = 55^\circ \\ 10 = 2x & m\angle C = m\angle D = 180 - 55 = 125^\circ \end{array}$$

Example 8: Given an isosceles trapezoid $HJKL$ with bases \overline{HJ} , \overline{LK} , and median \overline{RS} .

a. If $LK = 30$ and $HJ = 42$ find RS .

$$RS = \frac{1}{2}(LK + HJ) = \frac{1}{2}(42 + 30) = 36$$



b. If $RS = 17$ and $HJ = 14$ find LK .

$$\begin{array}{l} 2 \cdot 17 = \frac{1}{2}(x + 14) \\ 34 = x + 14 \quad x = 20 \quad LK = 20 \end{array}$$

c. If $RS = x + 5$ and $HJ + LK = 4x + 6$ find RS .

$$\begin{array}{l} x + 5 = \frac{1}{2}(4x + 6) \\ x + 5 = 2x + 3 \\ 5 = x + 3 \\ x = 2 \\ RS = 2 + 5 = 7 \end{array}$$

SUMMARY CHARTS:

Special Quadrilateral	Diagonals Are Always		Diagonals Always Bisect	
	Congruent	Perpendicular	Each Other	Angles
Parallelogram	No	No	Yes	No
Rectangle	Yes	No	Yes	No
Rhombus	No	Yes	Yes	Yes
Square	Yes	Yes	Yes	Yes
Trapezoid	No	No	No	No
Isosceles Trapezoid	Yes	No	No	No

Property	Rectangle	Rhombus	Square
1. All the properties of a parallelogram?	Yes	Yes	Yes
2. Equiangular (4 right angles?)	Yes	No	Yes
3. Equilateral (4 congruent sides?)	No	Yes	Yes
4. Diagonals bisect angles?	No	Yes	Yes
5. Diagonals congruent?	Yes	No	Yes
6. Diagonals perpendicular?	No	Yes	Yes