Math 1312
Section 5.1
Ratios, Rates, and Proportions

Definition:
A **ratio** is the quotient $\frac{a}{b}$, where $b \neq 0$ that provides comparison between the numbers a and b. Units of measure found in a ratio must be convertible to the same unit of measure.

Example 1: The ratio of two numbers (a and b) may be written in a variety of ways.

$$\frac{a}{b} \quad a \div b \quad a \text{ to } b \quad a : b$$

In writing the ratio of two numbers, it is usually helpful to express the ratio (fraction) in simplest form.

Example 2: $\frac{50}{100} = \frac{1}{2}$

Example 3: Find the best form of each ratio:

a) $\frac{8}{4}$
b) $\frac{8}{12}$
c) $\frac{4m}{60cm}$

Definition:
A **rate** is a quotient, that compares two quantities that cannot be converted to the same unit of measure.

Example 4: $\frac{60 miles}{3 gallons}$ $\frac{12 teaspoons}{2 quarts}$

Definition:
An equation that states that two ratios are equal is called a **proportion**.

$$\frac{a}{b} = \frac{c}{d}$$

The first and last terms (a and d) of the proportion are the **extremes**.
The second and third terms are the **means**.

Property 1: (Means - Extremes Property)
In a proportion, the product of the means equals the product of the extremes.

If $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$ and $d \neq 0$, then $a \times d = b \times c$

Example 5: Use the means-extremes property to solve each proportion for *x*.

a) $\frac{x}{8} = \frac{5}{12}$

b) $\frac{x}{20} = \frac{5}{x}$

c) $\frac{x+2}{5} = \frac{4}{x+1}$

Property 2: In a proportion, the means or the extremes (or both) may be interchanged.
In a proportion, the product of the means equals the product of the extremes.

If $\frac{a}{b} = \frac{c}{d}$ ($a \neq 0, b \neq 0, c \neq 0$, and $d \neq 0$), then $\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, and $\frac{d}{c} = \frac{b}{a}$

Example 6: Use Property 2 to rewrite $\frac{7}{8} = \frac{5}{12}$.

Property 3: If $\frac{a}{b} = \frac{c}{d}$ ($b \neq 0$ and $d \neq 0$), then $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a-b}{b} = \frac{c-d}{d}$.

Example 7: Use Property 3 to rewrite $\frac{2}{5} = \frac{7}{12}$.

Definition:
An **extended ratio** compares more that two quantities and is expressed in a form

$$a : b : c : d.$$

Example 8: The angles of a triangle are 60°, 90°, and 30°. Write the ratio that compares these measures.

Property 4: Unknown quantities in the ratio *a : b : c : d* should be represented by *ax*, *bx*, *cx*, and *dx*.

Example 9: The measures of two complementary angles are in the ratio 4 : 5. Find the measure of each angle.

Example 10: A recipe calls for 4 eggs and 3 cups of milk. To prepare for a larger number of guests, a cook uses 14 eggs. How many cups of milk are needed?