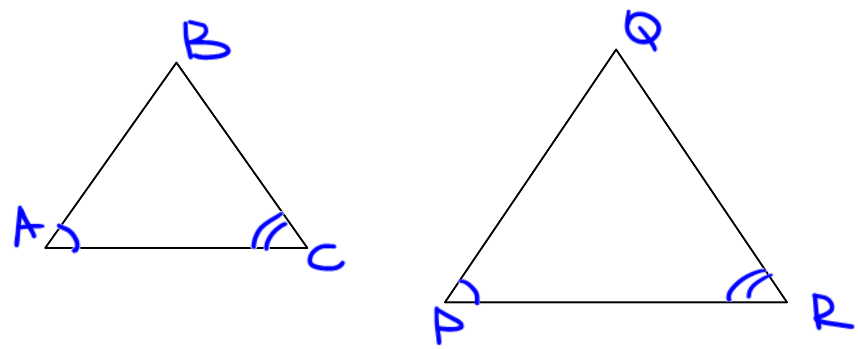
Math 1312
Section 5.3
Proving Triangles Similar

Postulate:
If three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar **(AAA)**.

The following corollaries of AAA Postulate are can be applied to help determine if two given triangles are similar:

Corollary 1 (AA):
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example 1:
Given: $\angle A \cong \angle P$,
$\angle C \cong \angle R$

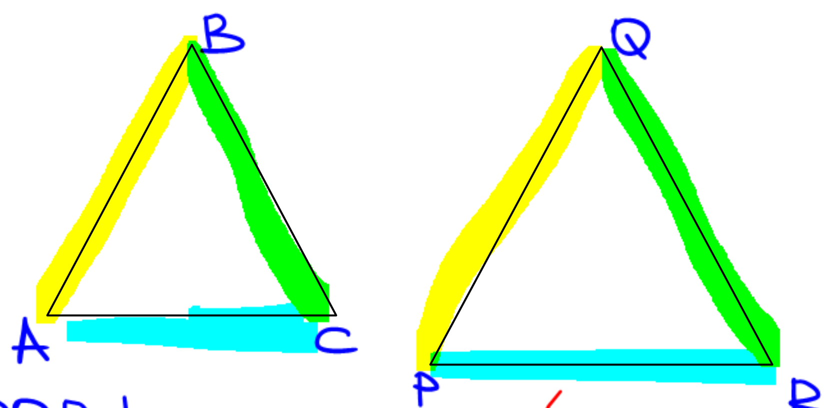


Conclusion:

$\triangle ABC \sim \triangle PQR$ by AA~

Corollary 2 (SSS): SSS ~
If each side of one triangle and the corresponding side of another triangle are proportional, then the triangles are similar.

Example 2:
Given: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



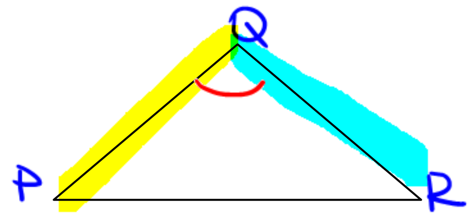
Conclusion:

$\triangle ABC \sim \triangle PQR$ by SSS ~ { SSS ≅

Corollary 3 (SAS): SAS~

If the measures of two sides of one triangle are proportional to the corresponding sides of another triangle AND the included angles are congruent, then the triangles are similar.

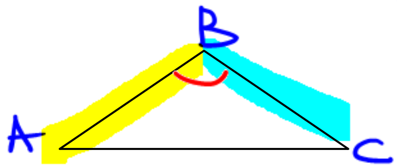
Example 3:



Given: $\frac{AB}{PQ} = \frac{BC}{QR}$,

$\angle B \cong \angle Q$

Conclusion:

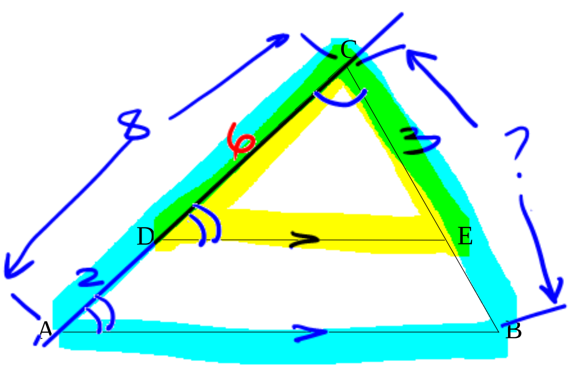


$\triangle ABC \sim \triangle PQR$

by SAS~

Example 4:

In the figure below, $\overline{AB} \parallel \overline{DE}$, $DA = 2$, $CA = 8$, and $CE = 3$. Find CB.



$\triangle CDE \sim \triangle CAB$

AA~ $\left[\begin{array}{l} \angle C \cong \angle C \\ \angle CDE \cong \angle CAB \text{ (as corr. } \angle s \text{ with } \parallel \text{ lines)} \end{array}\right.$

$\frac{\text{small}}{\text{big}}$

$\frac{CD}{CA} = \frac{CE}{CB}$

$\frac{6}{8} = \frac{3}{CB}$

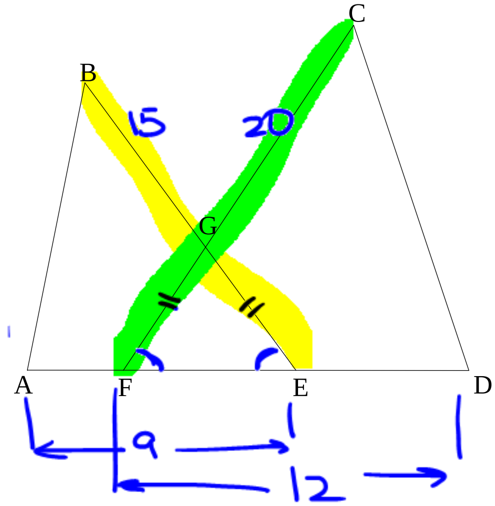
$6CB = 3(8)$

$CB = \frac{3(8)}{6}$

$\boxed{CB = 4}$

Example 5:

In the figure below, $\overline{FG} \cong \overline{EG}$, $BE = 15$, $CF = 20$, $AE = 9$, $DF = 12$. Determine which triangles in the figure are similar.



~~AA~~~

SSS~

$\boxed{\text{SAS}\sim}$

$\triangle ABE \sim \triangle DCF$

$\angle BEA \cong \angle CFD$

$\frac{AE}{DF} = \frac{BE}{CF}$?

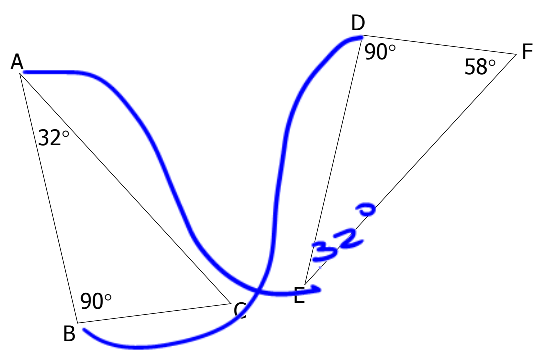
$\frac{9}{12} \cancel{=} \frac{15}{20}$

$\frac{3}{4} = \frac{3}{4}$ ✓

Example 6:

Determine whether each pair of triangles is similar. If so, write a mathematical sentence and give a reason that justifies your decision.

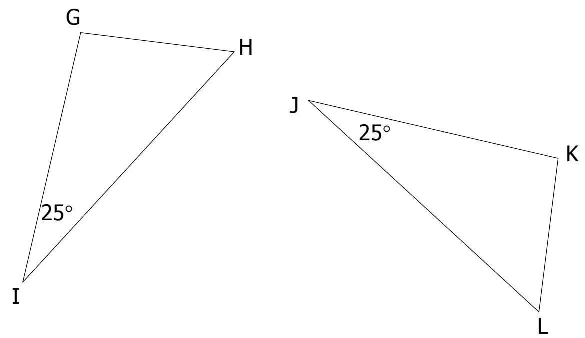
a)



$m\angle E = 180 - 90 - 58 = 32$

By AA~

b)



$\angle I \cong \angle J$

NO

CSSTP: Corresponding sides of similar triangles are proportional.

CASTC: Corresponding angles of similar triangles are congruent.

Theorem:

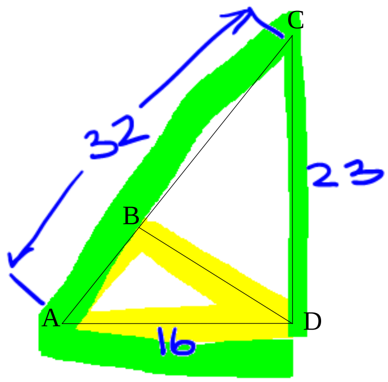
The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

Rules:

1. If two triangles are similar, then the **perimeters are proportional** to the measures of corresponding sides.
2. If two triangles are similar, then the measures of the corresponding **altitudes** (form 90°) **are proportional** to the measures of the corresponding sides.
3. If two triangles are similar, then the measures of the **corresponding angle bisectors** of the triangles **are proportional** to the measures of the corresponding sides.
4. If two triangles are similar, then the measures of the corresponding **medians are proportional** to the measures of the corresponding sides.

Example 7:

$\Delta ABD \sim \Delta ADC$. If $AD = 16$, $AC = 32$, and $DC = 23$ find the perimeter of ΔABD.



$P(\Delta ADC) = 16 + 23 + 32 = 71$

$$\frac{P(\Delta ADC)}{P(\Delta ABD)} = \frac{AC}{AD}$$

$$\frac{71}{P(\Delta ABD)} = \frac{32}{16}$$

$$\frac{71}{P(\Delta ABD)} = 2$$

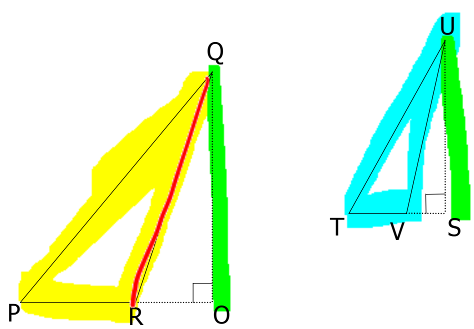
$$P(\Delta ABD) = \frac{71}{2} = \boxed{35.5}$$

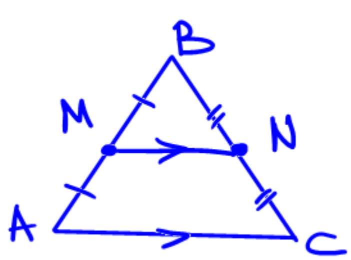
Example 8:

ΔPQR ~ ΔTUV. IF QO is an altitude of ΔPQR, and US is an altitude of ΔTUV, then complete the following:

$$\frac{QO}{US} = \frac{QR}{?}$$

UV





1. $\overline{MN} \parallel \overline{AC}$
2. $MN = \frac{1}{2} AC$

Lemma:
If a line segment divides two sides of a triangle proportionally, then the line segment is parallel to the third side of the triangle.

Converse:

If we are given ΔABC and ΔDEC, where $\overline{DE} \parallel \overline{AB}$.

Because DE ∥ AB, we can conclude that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$ (corresponding angles are congruent).

This makes $\Delta ABC \sim \Delta DEC$ by the AA similarity property.

So, now we can conclude that $\frac{CD}{CA} = \frac{CE}{CB}$.