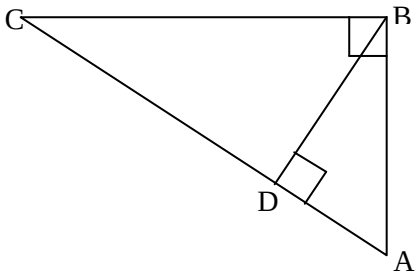
Math 1312
Section 5.4
The Pythagorean Theorem

Review of radicals

Example 1: Simplify $\sqrt{80}$

Example 2: Simplify $\sqrt{\frac{7}{2}}$

Theorem 1: The altitude drawn to the hypotenuse of a right triangle separates the triangle into two right triangles that are similar to each other and to the original triangle.



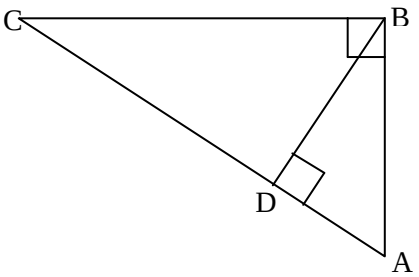
Example 3: Name the similar triangles in the figure above.

Definition: The geometric mean between two positive numbers, a and b, is the positive number, x, where: $\frac{a}{x} = \frac{x}{b}$

Theorem 2: The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of hypotenuse.

$$\frac{part}{altitude} = \frac{altitude}{PART}$$

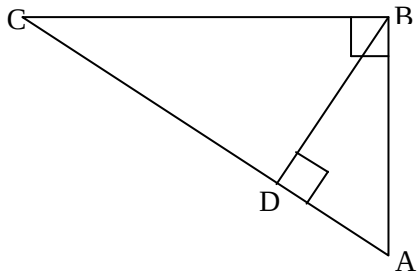
$$\frac{AD}{BD} = \frac{BD}{DC}$$



Theorem 3: The length of each leg of a right triangle is the geometric mean of the length of the hypothenuse and the length of the segment of the hypotenuse adjacent (next) to that leg.

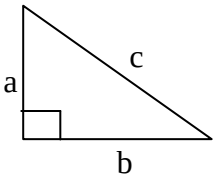
$$\frac{part}{leg} = \frac{leg}{whole}$$

$$\frac{AD}{AB} = \frac{AB}{AC} \text{ or } \frac{DC}{BC} = \frac{BC}{AC}$$



Pythagorean Theorem: The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.

$$a^2 + b^2 = c^2$$



Question 1: What does it mean when $a^2 + b^2 > c^2$?

Question 2: What does it mean when $a^2 + b^2 < c^2$?

Example 4: Determine the type of triangle if the lengths of its sides are:

a) 4, 3, 5

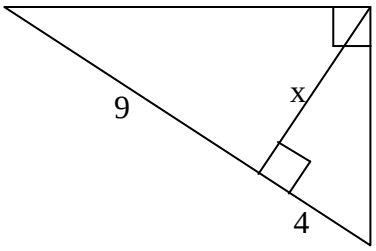
b) 4, 5, 6

c) 3, 4, 9

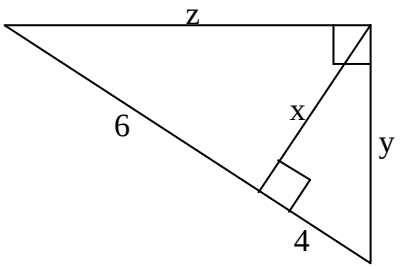
d) 4, 5, 7

Example 5: Find the geometric mean between 4 and 18.

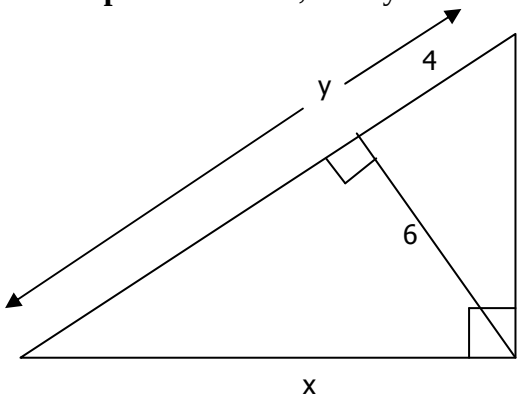
Example 6: Find “x”.



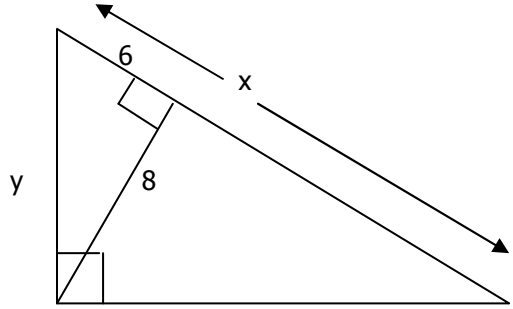
Example 7: Find “x”, “y”, and “z”.



Example 8: Find “x”, and “y”.

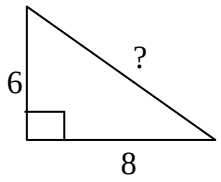


Example 9: Find “x”, and “y”.



Example 10: Find the “missing” length.

a)



b)