Math 1312
Section 5.4
The Pythagorean Theorem

Review of radicals

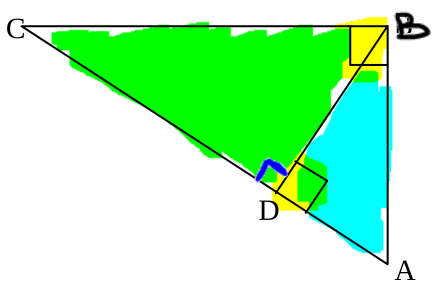
$4=2^2$
$9=3^2$
$16=4^2$
$25=5^2$
$36=6^2$
$49=7^2$
$64=8^2$
$81=9^2$
$100=10^2$

Example 1: Simplify $\sqrt{80}$

$= \sqrt{16\cdot 5} = \sqrt{16}\cdot\sqrt{5} = \boxed{4\sqrt{5}}$

Example 2: Simplify $\sqrt{\frac{7}{2}}$ $= \frac{\sqrt{7}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \boxed{\frac{\sqrt{14}}{2}}$

Theorem 1: The altitude drawn to the hypotenuse of a right triangle separates the triangle into two right triangles that are similar to each other and to the original triangle.



Example 3: Name the similar triangles in the figure above.

$\triangle CBA \sim \triangle CDB \sim \triangle BDA$

Definition: The geometric mean between two positive numbers, a and b, is the positive number, x, where: $\frac{a}{x} = \frac{x}{b}$

$\sqrt{x^2} = \sqrt{ab}$ $\boxed{x = \sqrt{ab}}$

$5, 7$ $\frac{5+7}{2} = 6$

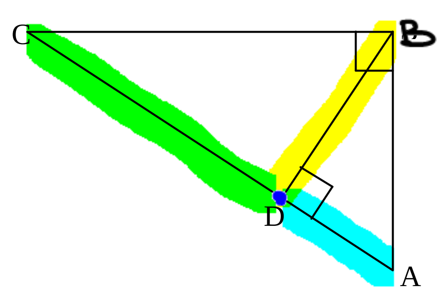
$\sqrt{5\cdot 7} = \sqrt{35}$

Theorem 2: The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of hypotenuse.

$$\frac{part}{altitude} = \frac{altitude}{PART}$$

$\boxed{BD = \sqrt{CD\cdot DA}}$

$$\frac{AD}{BD} = \frac{BD}{DC}$$



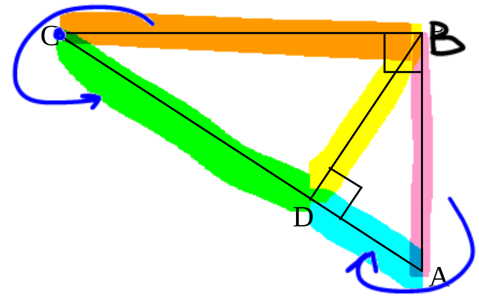
Theorem 3: The length of each leg of a right triangle is the geometric mean of the length of the hypothenuse and the length of the segment of the hypotenuse adjacent (next) to that leg.

$$\frac{part}{leg} = \frac{leg}{whole}$$

$BC = \sqrt{AC \cdot CD}$

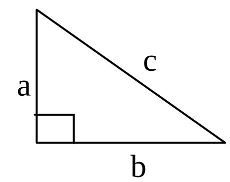
$AB = \sqrt{AC \cdot AD}$

$$\frac{AD}{AB} = \frac{AB}{AC} \quad \text{or} \quad \frac{DC}{BC} = \frac{BC}{AC}$$



Pythagorean Theorem: The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.

$$\boxed{a^2 + b^2 = c^2}$$



Question 1: What does it mean when $a^2 + b^2 > c^2$?

acute

Question 2: What does it mean when $a^2 + b^2 < c^2$?

obtuse

Example 4: Determine the type of triangle if the lengths of its sides are:

a) 4, 3, 5

① $4+3>5$ ✓ ② $4^2+3^2=5^2$

$25=25$

right

b) 4, 5, 6

① $4+5>6$ ✓ ② $4^2+5^2>6^2$

$16+25>36$

$41>36$

acute

c) 3, 4, 9

① $3+4 \not> 9$

not a $\triangle$

d) 4, 5, 7

① $4+5>7$ ② $4^2+5^2<7^2$

$41<49$

obtuse

Example 5: Find the geometric mean between 4 and 18.

$$x = \sqrt{4\cdot 18} = \sqrt{72} = \sqrt{9\cdot 8} = \sqrt{9}\cdot\sqrt{8} = 3\sqrt{8}$$

$$= 3\sqrt{4\cdot 2} = 3\sqrt{4}\cdot\sqrt{2}$$

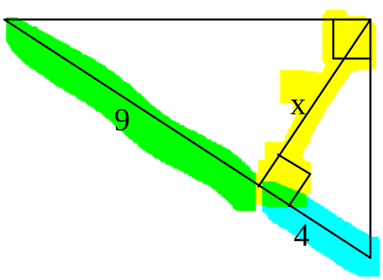
$$= 3(2)\sqrt{2}$$

$$= \boxed{6\sqrt{2}}$$

$$= \sqrt{72} = \sqrt{36\cdot 2}$$

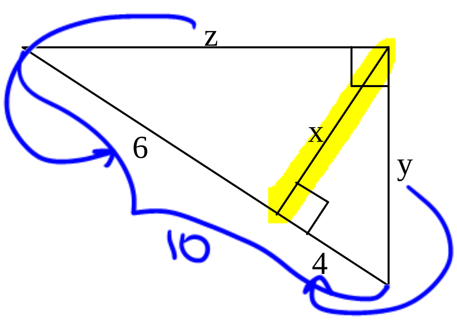
$$= \sqrt{36}\cdot\sqrt{2} = \boxed{6\sqrt{2}}$$

Example 6: Find "x".



$$x = \sqrt{9\cdot 4} = \sqrt{36} = \boxed{6}$$

Example 7: Find "x", "y", and "z".



$$x = \sqrt{6\cdot 4} = \sqrt{6}\cdot\sqrt{4} = \boxed{2\sqrt{6}}$$

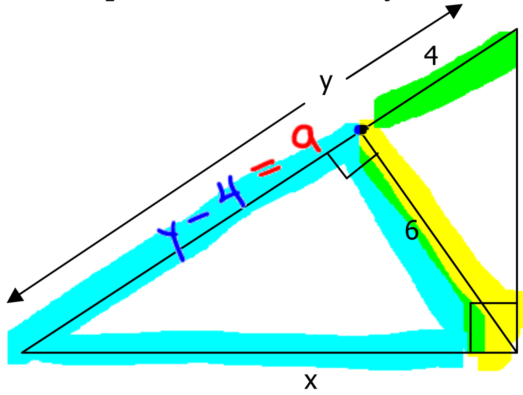
$$y = \sqrt{10\cdot 4} = \sqrt{10}\cdot\sqrt{4} = \boxed{2\sqrt{10}}$$

$$z = \sqrt{10\cdot 6} = \sqrt{60} = \sqrt{4\cdot 15}$$

$$= \sqrt{4}\cdot\sqrt{15}$$

$$= \boxed{2\sqrt{15}}$$

Example 8: Find “x”, and “y”.



$$(6)^2 = \left(\sqrt{4(y-4)}\right)^2$$

$$\frac{\cancel{36}\,9}{\cancel{4}} = \frac{\cancel{4}(y-4)}{\cancel{4}}$$

117

1+1+7 = 9

$$9 = y-4$$

$$\boxed{y = 13}$$

$$x^2 = 6^2 + 9^2$$

$$x^2 = 36 + 81$$

$$\sqrt{x^2} = \sqrt{117}$$

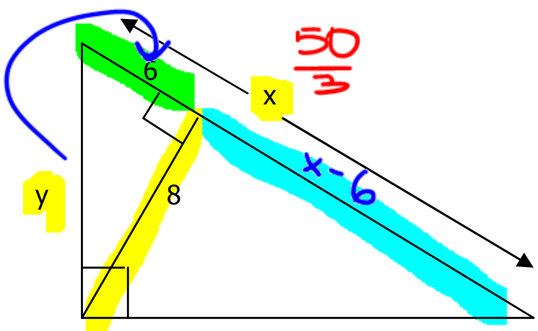
$$x = \sqrt{117}$$

$$= \sqrt{9 \cdot 13}$$

$$x = \sqrt{9} \cdot \sqrt{13}$$

$$\boxed{x = 3\sqrt{13}}$$

Example 9: Find “x”, and “y”.



$$(8)^2 = \left(\sqrt{6(x-6)}\right)^2$$

$$64 = 6(x-6)$$

$$64 = 6x - 36$$

$$+36 \qquad +36$$

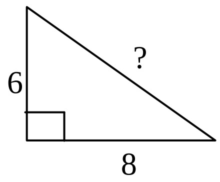
$$\frac{100}{6} = \frac{6x}{6}$$

$$x = \frac{100}{6} = \frac{50}{3} = 16\frac{2}{3}$$

$$y = \sqrt{\frac{50}{\cancel{3}} \cdot \cancel{6}^2} = \sqrt{100} = \boxed{10}$$

Example 10: Find the "missing" length.

a)

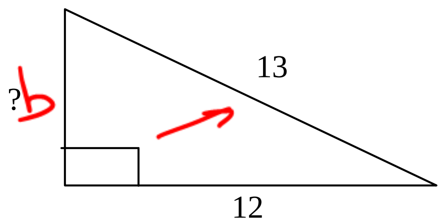


$a^2+b^2=c^2$

$\sqrt{6^2+8^2}=\sqrt{c^2}$

$\boxed{c=10}$

b)



$a^2+b^2=c^2$

$12^2+b^2=13^2$

$144+b^2=169$

$-144 \qquad -144$

$\sqrt{b^2}=\sqrt{25}$

$\boxed{b=5}$