Math 1312
Section 5.5
Special Right Triangles

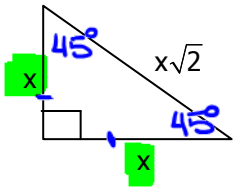
Note: Triangles in this section are always right triangles!

45-45-90 Triangles

Theorem 1: In a triangle whose angles measure 45^0, 45^0, and 90^0, the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

The ratio of the sides of a 45-45-90 triangle are: $x : x : x\sqrt{2}$.

$1 : 1 : \sqrt{2}$



45-45-90

if this measure is given:	*and you want this measure:*	*then do this:*
the leg	hypotenuse	multiply the leg by $\sqrt{2}$
hypotenuse	the leg	divide hypotenuse by $\sqrt{2}$

Note: The legs of a 45-45-90 are the same measure!

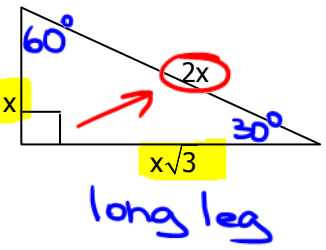
$$leg = \frac{hypotenuse}{\sqrt{2}} \qquad hypotenuse = leg\sqrt{2}$$

30-60-90 Triangles

Theorem 2: In a triangle whose angles measure 30^0, 60^0, and 90^0, the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ And the length of the shorter leg.

The ratio of the sides of a 30-60-90 triangle are: $x : x\sqrt{3} : 2x$.

$1 : \sqrt{3} : 2$ short leg



Note: The short leg is always opposite the 30° angle!

It is best to find the measure of the short leg first (that is if it is not given).

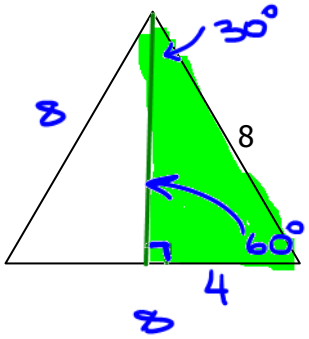
30-60-90

if this measure is given:	*and you want this measure:*	*then do this:*
short leg	hypotenuse	multiply short leg by 2
short leg	long leg	multiply short leg by $\sqrt{3}$
long leg	short leg	divide long leg by $\sqrt{3}$
hypotenuse	short leg	divide hypotenuse by 2

$$short\_leg = \frac{hypotenuse}{2} \qquad short\_leg = \frac{long\_leg}{\sqrt{3}}$$

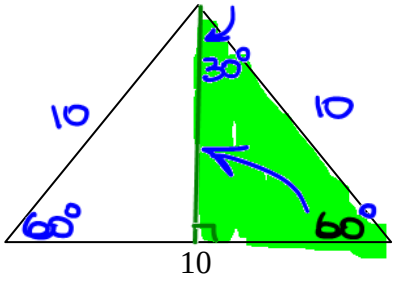
$$hypotenuse = 2(short\_leg) \qquad long\_leg = \sqrt{3}(short\_leg)$$

Example 1: Find the altitude of the following equilateral triangle.



Altitude = Long Leg
= Short Leg · $\sqrt{3}$
= $4\sqrt{3}$

Example 2: Find the altitude of the following equilateral triangle.



30-60-90 Δ

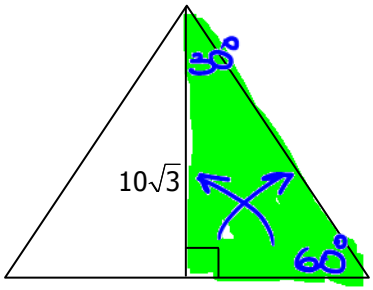
Hyp = 10 Short Leg = $\frac{10}{2} = 5$

altitude = Long Leg

$= SL \cdot \sqrt{3}$

$= \boxed{5\sqrt{3}}$

Example 3: Find the length of the side of the following equilateral triangle.



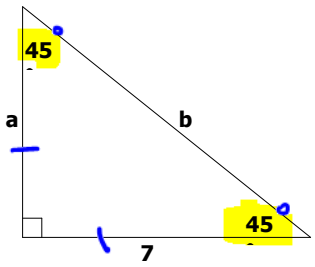
Side = hypotenuse

Long Leg = $10\sqrt{3}$

Short Leg = $\frac{LL}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$

Hyp = $2 \cdot SL = 2(10) = \boxed{20}$

Example 4: Find a and b.

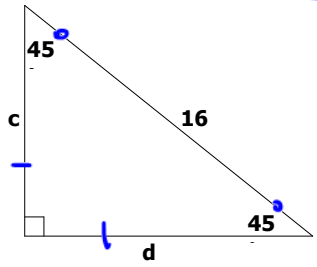


45-45-90 Δ

$a = 7$

$b = 7\sqrt{2}$

Example 5: Find c and d.



45-45-90 Δ

$$c = d = \frac{Hyp}{\sqrt{2}} = \frac{16 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{16\sqrt{2}}{2}$$

$$= 8\sqrt{2}$$

Popper #14

5 Ds