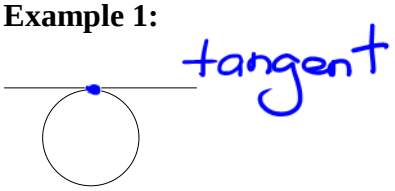
Math 1312
Section 6.2
More Angle Measures in Circle

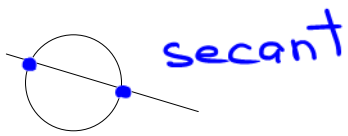
Definitions:

A **tangent** is a line that intersects a circle at exactly one point (**point of contact** or **point of tangency**).

A **secant** is a line (or a segment or ray) that intersects a circle at exactly two points.

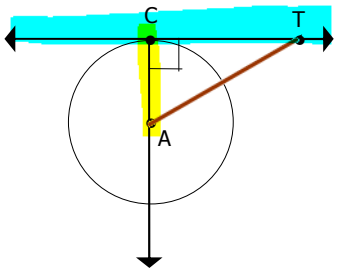
Example 1:





Theorem 1:
The radius drawn to a tangent at the point of tangency is perpendicular to the tangent at that point.

Example 2:



Given: $\odot A$
$\overline{CA}$ is a radius
$\overleftrightarrow{CT}$ is a tangent
Concl: $\overleftrightarrow{CT} \perp \overline{CA}$

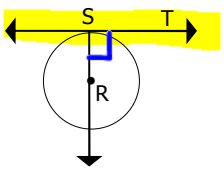
Note: Because $\angle TCA$ is a right angle then ΔTCA is a right triangle. Therefore, you could use the Pythagorean theorem to find the measure of a missing side.

$AT^2 = AC^2 + CT^2$

Theorem 2 (converse of Theorem 1)**:**

If a line is perpendicular to a radius then the line is a tangent of the circle.

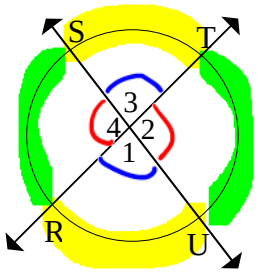
Example 3:

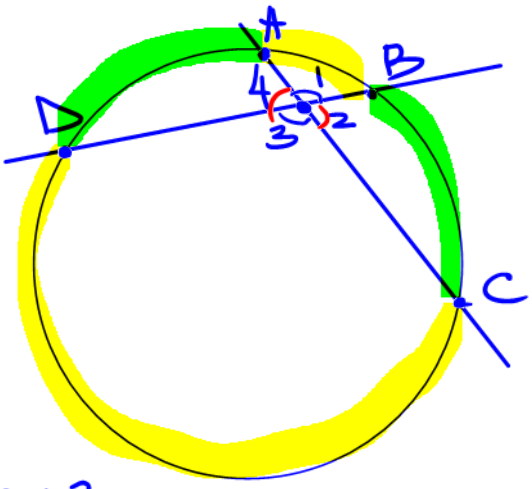


Given: $\overleftrightarrow{ST} \perp \overline{SR}$ &
$\overline{SR}$ is a radius
Concl: $\overleftrightarrow{ST}$ is a tangent

Theorem 3:
The measure of an angle formed by two chords (two secants) that intersect within a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

Example 4:





$m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{RU} + m\overset{\frown}{ST})$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{RS} + m\overset{\frown}{TU})$

$m\angle 1 = m\angle 3$

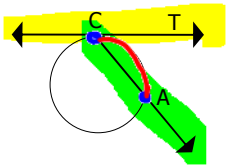
$m\angle 2 = m\angle 4$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$

Theorem 4:
The measure of an angle formed by a tangent and a chord drawn to the point of tangency (a tangent and a secant) is one-half the measure of the intercepted arc.

Example 5:

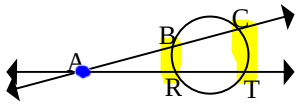


$m\angle TCA = \frac{1}{2}m\overset{\frown}{CA}$

Theorem 5:
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

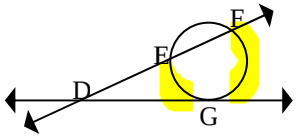
Example 6:

There are three possible cases:



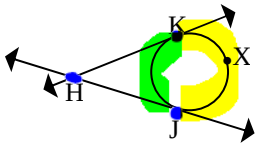
Two Secants

$$m\angle CAT = \frac{1}{2}(\overset{\frown}{mCT} - \overset{\frown}{mBR})$$



A Secant and a Tangent

$$m\angle FDG = \frac{1}{2}(\overset{\frown}{mFG} - \overset{\frown}{mEG})$$



Two Tangents

$$m\angle KHJ = \frac{1}{2}(\overset{\frown}{mKXJ} - \overset{\frown}{mKJ})$$

In Summary:

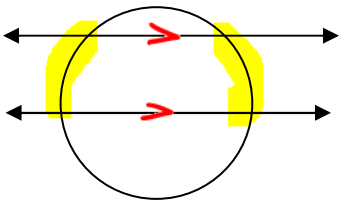
If the lines intersect **ON** the circle use: angle = $\frac{1}{2}$(arc)

If the lines intersect **IN** the circle use: angle = $\frac{1}{2}$(arc + arc)

If the lines intersect **OUT** of the circle use: angle = $\frac{1}{2}$(big arc – little arc)

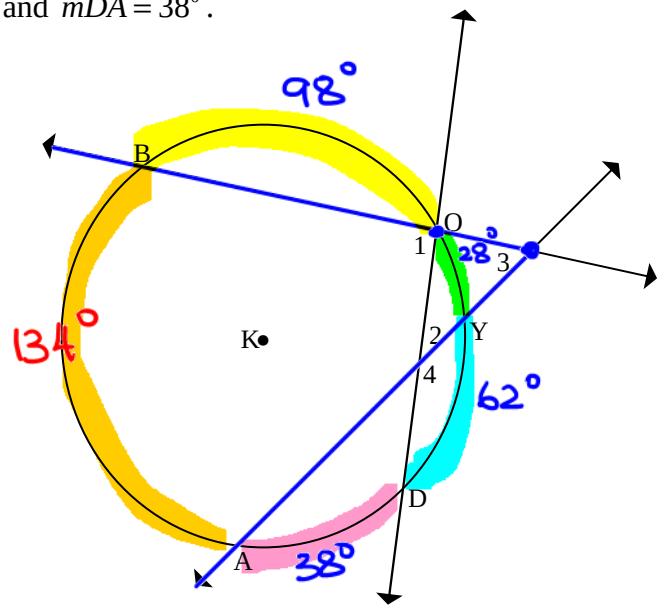
Theorem 6:
If two parallel lines intersect a circle, the intercepted arcs between those lines are congruent.

Example 7:



Example 8:

In circle K, $m\overset{\frown}{OB} = 98^0$, $m\overset{\frown}{OY} = 28^0$, $m\overset{\frown}{YD} = 62^0$, and $m\overset{\frown}{DA} = 38^0$.
Find each measure.



a) $m\overset{\frown}{AB} = 360 - 98 - 28 - 62 - 38 = 134^\circ$

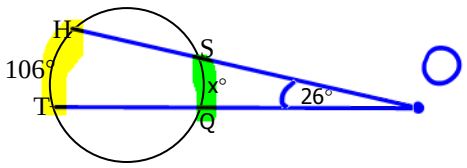
b) $m\angle 1 = \frac{1}{2}(134 + 38) = \frac{1}{2}(172) = 86^\circ$

c) $m\angle 2 = \frac{1}{2}(28 + 38) = \frac{1}{2}(66) = 33^\circ$

d) $m\angle 3 = \frac{1}{2}(134 - 28) = \frac{1}{2}(106) = 53^\circ$

e) $m\angle 4 = 180 - 33 = 147^\circ$

Example 9: Find the value of "x".



$m\angle HOT = \frac{1}{2}(m\overset{\frown}{HT} - m\overset{\frown}{SQ})$

$2 \cdot 26 = 2 \cdot \frac{1}{2}(106 - x)$

$52 = 106 - x$

$0 = 54 - x$

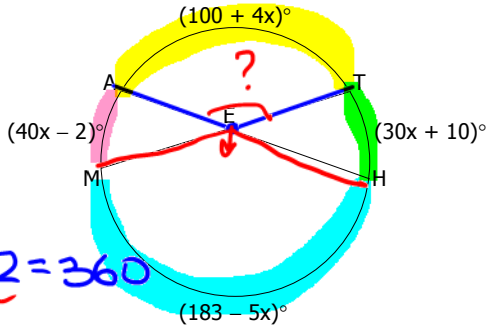
$x = 54$

$m\overset{\frown}{SQ} = 54^\circ$

Example 10: Refer to the circle below, find:

a) $x = 1$

b) $m\angle AET = \frac{1}{2}(104 + 178) = 141^\circ$



$100 + 4x + 30x + 10 + 183 - 5x + 40x - 2 = 360$

$69x + 291 = 360$

$69x = 69$

$x = 1$