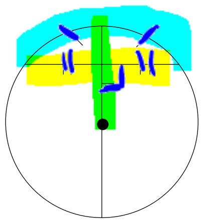
Math 1312
Section 6.3
Line and Segment Relationships in the Circle

Theorem 1: If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc.

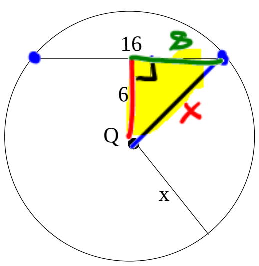
Example 1:



Theorem 2 (converse of Theorem 1)**:** If a line through the center of a circle bisects a chord other than a diameter, then it is perpendicular to that chord.

Theorem 3: The perpendicular bisector of a chord contains the center of the circle.

Example 2: Find the value of “x”.



$x^2 = 6^2 + 8^2$

$x^2 = 36 + 64$

$\sqrt{x^2} = \sqrt{100}$

$\boxed{x = 10}$

Definitions:

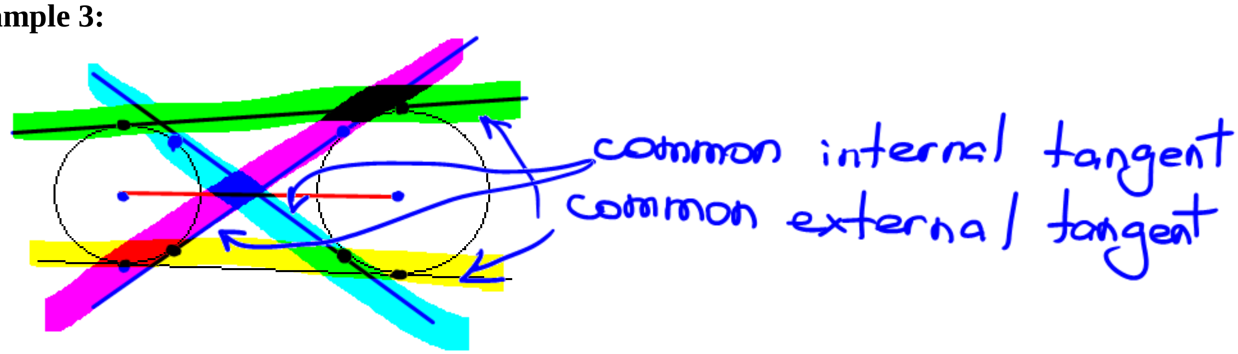
A **tangent** is a line that intersects a circle at exactly one point.

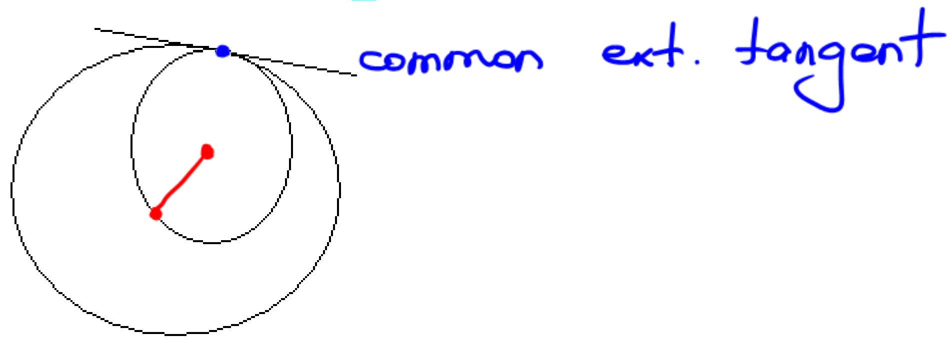
A line (or line segment) that is tangent to two circles is called a **common tangent** for these circles.

If a common tangent *does not* intersect the line of centers, it is a **common external tangent**.

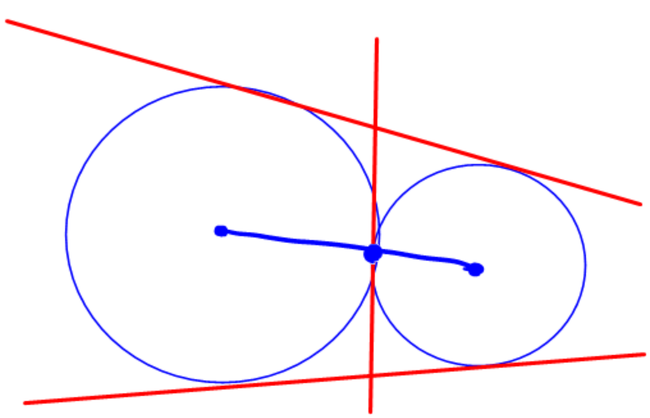
If a common tangent *does* intersect the line of centers, it is a **common internal tangent**.

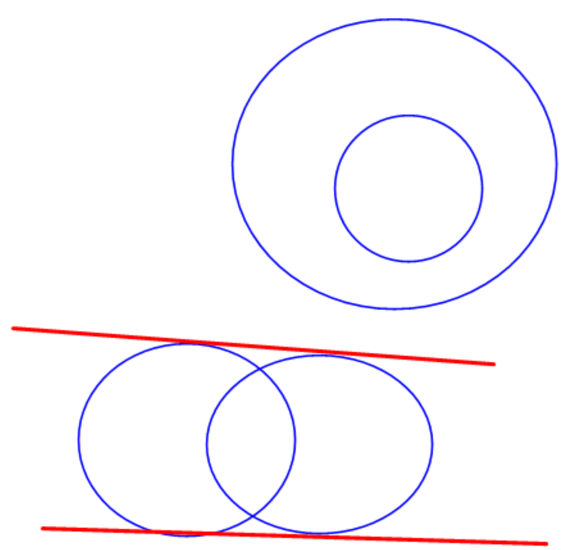
Example 3:





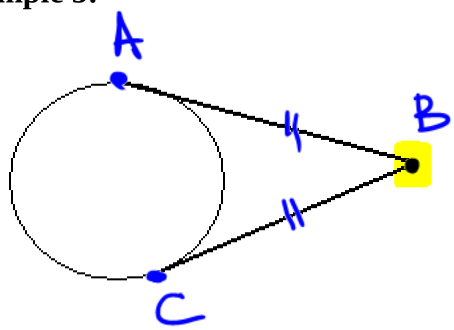
Example 4: Draw two circles that have exactly 3 common tangents.





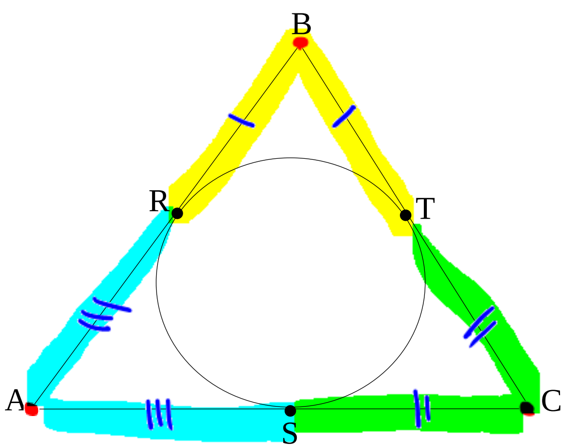
Theorem 4: The tangent segments to a circle from the same external point are congruent.

Example 5:



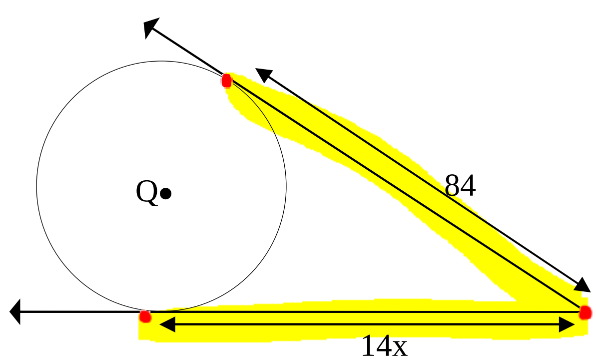
AB = BC

Example 6:



BR = BT
TC = CS
AR = AS

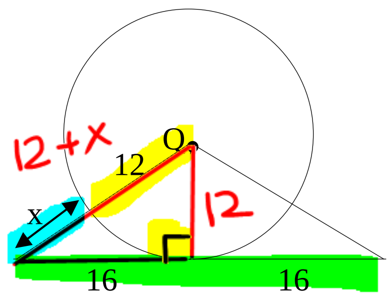
Example 7:



$14x = 84$

$\boxed{x = 6}$

Example 8:



$$(12+x)^2 = 12^2 + 16^2$$

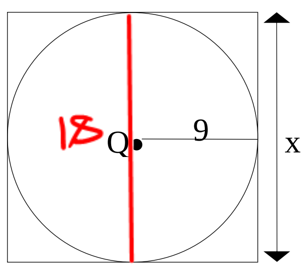
$$(12+x)^2 = 144 + 256$$

$$\sqrt{(12+x)^2} = \sqrt{400}$$

$$12 + x = 20$$

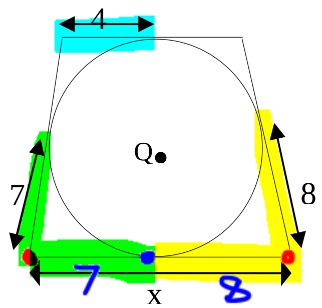
$$\boxed{x = 8}$$

Example 9:



$$\boxed{x = \text{diameter} = 18}$$

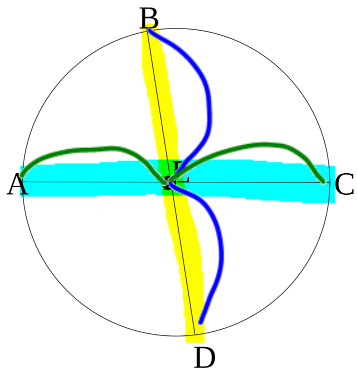
Example 10:



$$x = 7 + 8 = 15$$

Theorem 5: If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.

Example 11:

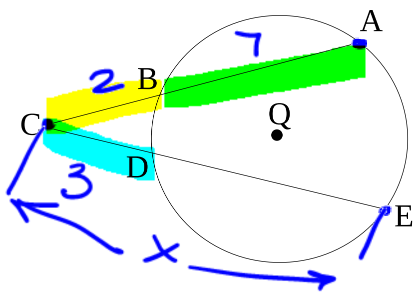


$\underbrace{AE \times EC}$ $=$ $\underbrace{BE \times ED}$

$(\, part \times part \,)$

Theorem 6: If two secant segments are drawn to a circle from an external point, then the products of the lengths of each secant with its external segment are equal.

Example 11:



$\underbrace{CA \times CB}$ $=$ $\underbrace{CE \times CD}$

$(\, whole \times exterior \,)$

$CB = 2$

$BA = 7$

$CD = 3$

Find $CE = x$

$9(2) = x(3)$

$18 = 3x$

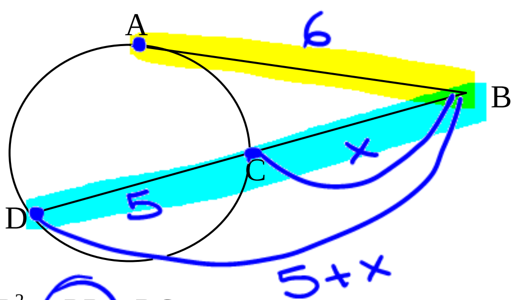
$\boxed{x = 6}$

Theorem 6: If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the length of the tangent segment is equal to the product of the length of the secant with its external segment.

Example 12:

$AB = 6$
$DC = 5$

Find $BC = x$



$AB^2 = BD \times BC$

$6^2 = (5+x)x$

$36 = 5x + x^2$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

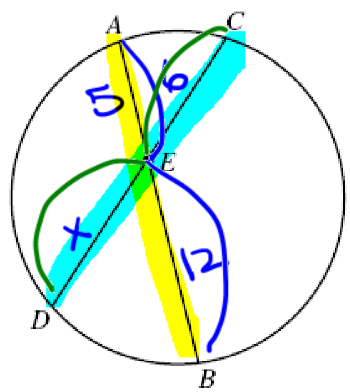
$x + 9 = 0$

$\cancel{x = -9}$

$x - 4 = 0$

$\boxed{x = 4}$

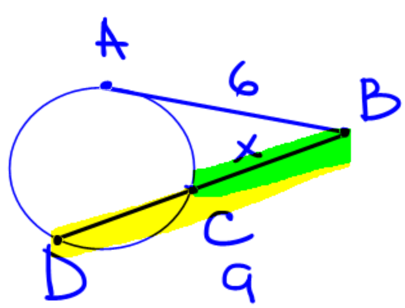
Example 13: In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at E. If $AE = 5$, $BE = 12$, and $CE = 6$, what is the length of $\overline{DE}$?



$5(12) = 6x$

$60 = 6x$

$\boxed{x = 10}$



$AB = 6$
$BD = 9$

Find $BC = x$

$AB^2 = BD(BC)$

$6^2 = 9x$

$36 = 9x$

$\boxed{x = 4}$

Popper 17

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