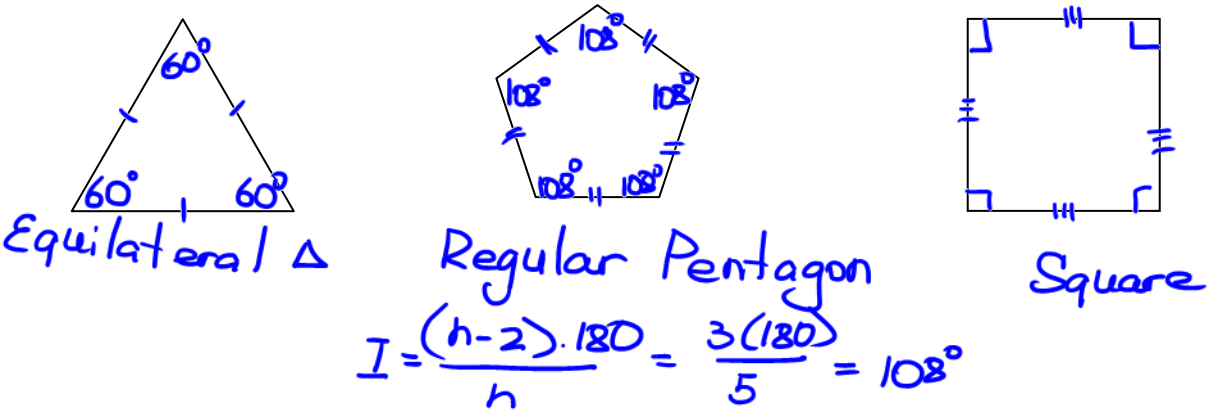
Math 1312
Section 8.3
Regular Polygons and Area

Definition:

A **regular polygon** is a polygon that is both equilateral (all sides are congruent) and equiangular (all angles are congruent).

Example1:



Theorem 1: The measure *I* of each interior angle of a regular polygon of *n* sides is

$$I = \frac{(n-2)\cdot 180^\circ}{n}.$$

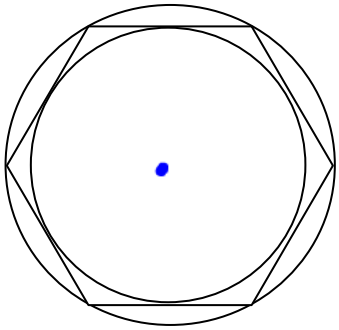
Definitions:

A polygon is **circumscribed about** a circle if all of its vertices lie on the circle.

A polygon is **inscribed in** a circle if each of its sides is tangent to the circle.

The **center of a regular polygon** is the common center for the inscribed and circumscribed circles of the polygon.

Example 2:



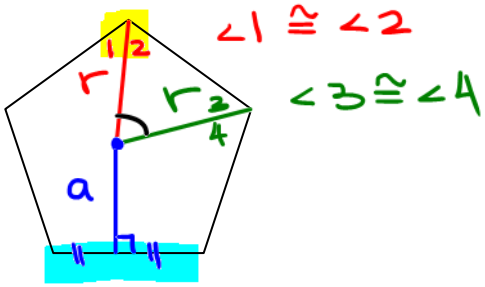
Definitions:

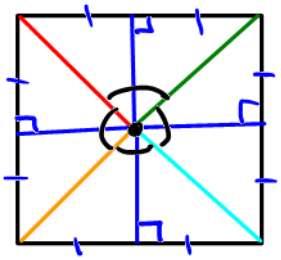
The segment from the center of a regular polygon perpendicular to a side of a regular polygon is called an **apothem**.

The segment from the center to a vertex a regular polygon is the **radius** of the regular polygon.

A **central angle** of a regular polygon is the angle formed by two consecutive radii.

Example 3:



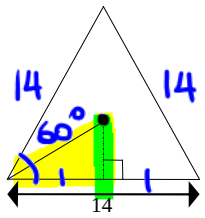


Theorem 2: The measure of each central angle is found by: $C = \frac{360}{n}$

Theorem 3: Any radius of a regular polygon bisects the angle at the vertex to which it is drawn and any apothem bisects the side to which it is drawn.

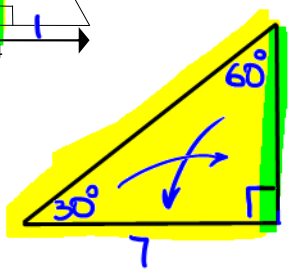
Theorem 4: The area of any regular polygon can be found by: $A = \frac{1}{2}aP$, where a = apothem and P = perimeter.

Example 4: Find the apothem (a), area (A), and perimeter (P) of each regular polygon.



$A = \frac{1}{2}aP$

$P = 3(14) = 42$



$LL = 7$

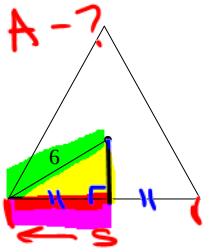
$a = SL = \frac{LL}{\sqrt{3}} = \frac{7 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{7\sqrt{3}}{3}$

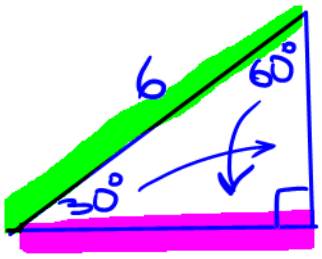
$A = \frac{1}{2}\left(\frac{7\sqrt{3}}{3}\right)(42) = 49\sqrt{3}$

P - ?

a - ?

A - ?





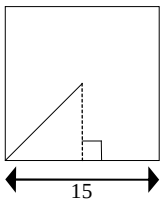
Apothem = Short Leg = $\frac{\text{Hyp}}{2} = \frac{6}{2} = \boxed{3}$

Side of Eq $\triangle$ = 2 Long Leg = $2(\text{SL}\sqrt{3})$

$= 2 \cdot 3\sqrt{3} = 6\sqrt{3}$

$P = 3(6\sqrt{3}) = 18\sqrt{3}$

$A = \frac{1}{2}aP = \frac{1}{2}(3)(18\sqrt{3}) = 27\sqrt{3}$ (a)



Example 5: Find the apothem (a), area (A), and perimeter (P) of each regular polygon.

a) Hexagon with $a=8$

b) Octagon with apothem = 4.8, side = 4

c) Square with apothem = 24