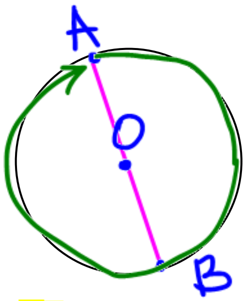
Math 1312
Section 8.4
Circumference and Area of a Circle

Definitions:

A **circle** (symbol $\odot$) is the set of all points in a plane that are at the same distance from the center.



$\odot O$

The **diameter** is a chord through the center of a circle.

The **diameter** is the distance across the circle.

The **circumference** of a circle is the distance around the circle.

Definition: π is a constant equal to 3.14 or 3.1416 or $\frac{22}{7}$.

Theorem 1: The circumference of a circle is given by the formula $\boxed{C = \pi d}$ or $\boxed{C = 2\pi r}$.

Definition: The length of an arc is the distance between the endpoints of the arc.

Theorem 2: In a circle whose circumference is C, the length ℓ of an arc whose degree measure is m is given by $\boxed{\ell = \frac{m}{360} \cdot C}$.

Theorem 3: The area A of a circle whose radius has length r is given by $\boxed{A = \pi r^2}$.

Example 1: Find the diameter, circumference, and the area of a circle whose radius is 8 cm.

$$d = 2r = 2(8) = 16 \text{ cm}$$

$$C = \pi d = \pi(16) = 16\pi \text{ cm}$$

$$A = \pi r^2 = \pi(8)^2 = 64\pi \text{ cm}^2$$

Example 2: Find the radius, the diameter, and the area of a circle whose circumference is 22π in.

$C = 2\pi r$

$22\cancel{\pi} = 2\cancel{\pi} r$

$22 = 2r$

$\boxed{r = 11 \text{ in}}$

$d = 2(11) = \boxed{22 \text{ in}}$

$A = \pi r^2 = \pi(11)^2 = \boxed{121\pi \text{ in}^2}$

Example 3: Find the radius and circumference of a circle whose area is $49\pi\ m^2$.

$A = \pi r^2$

$49\cancel{\pi} = \cancel{\pi} r^2$

$\sqrt{49} = \sqrt{r^2}$

$\boxed{r = 7 \text{ m}}$

$C = 2\pi r = 2\pi(7) = \boxed{14\pi \text{ m}}$

Example 4: Find the length of a 48^0 arc in a circle whose diameter is 14. in

$\ell = \frac{m}{360} \cdot C = \frac{48}{360} \cdot 14\pi = \boxed{1.87\pi \text{ in}}$

$C = \pi d = \pi(14) = 14\pi$

Example 5: Find the length of a 72^0 arc in a circle whose circumference is 45π.

$\ell = \frac{m}{360} \cdot C = \frac{72}{360} \cdot 45\pi = \boxed{9\pi}$

Example 6: Find the radius of a circle if a 90^0 arc has length of 6π.

$\ell = \frac{m}{360} \cdot C$

$C = 2\pi r$

$6\pi = \frac{\cancel{90}\,1}{\cancel{360}\,4} \cdot C$

$4 \cdot 6\pi = \cancel{4} \cdot \frac{1}{\cancel{4}} C$

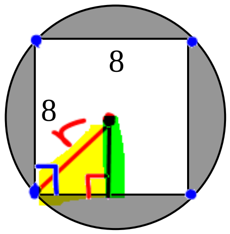
$24\pi = C$

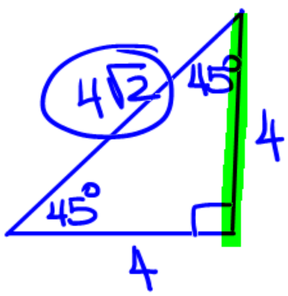
$24\cancel{\pi} = 2\cancel{\pi} r$

$24 = 2r$

$\boxed{r = 12}$

Example 7: Find the exact area of the shaded region.





$A_{shaded} = A_{circle} - A_{square}$

$A_{square} = (8)^2 = 64$

$A_{circle} = \pi r^2 = \pi (4\sqrt{2})^2$

$= \pi (16)(2)$

$= 32\pi$

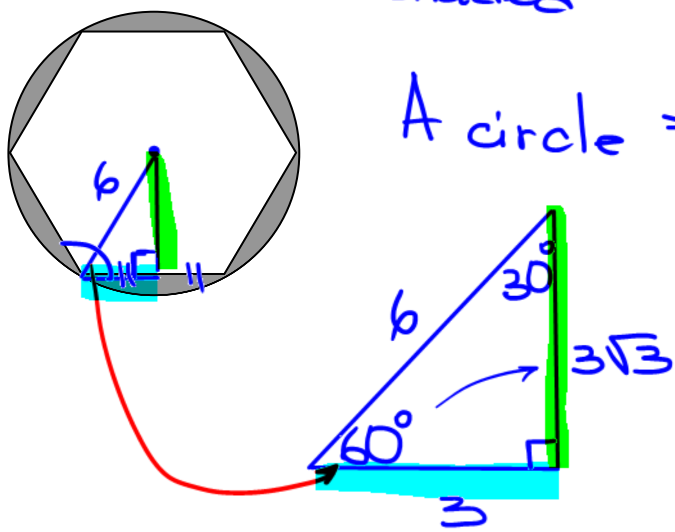
$A_{shaded} = \boxed{32\pi - 64}$

Example 8: Find the exact area of the shaded region (regular hexagon is inscribed in a circle of radius 6).

$= \boxed{36\pi - 54\sqrt{3}}$

$A_{shaded} = A_{circle} - A_{hexagon}$

$A_{circle} = \pi r^2 = \pi (6)^2 = 36\pi$



$A = \frac{1}{2} a P$

$A = \cancel{\frac{1}{2}}\, 3\sqrt{3} \left(\overset{18}{\cancel{36}}\right)$

$= 54\sqrt{3}$

$P = 6\,\text{side} = 6(3 \cdot 2) = 36$

Popper 20

A
B
C
D
E
A
B
C
D
E