Math 1313 Final Exam Review

Example 1: Find the equation of the line containing points (1,2) and (2,3).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{2-1} = \frac{1}{1}$

$y = mx + b$
$2 = (1)(1) + b$
$2 = 1 + b$
$1 = b$
$y = x + 1$

$y - y_1 = m(x - x_1)$
$y - 2 = 1(x - 1)$
$y - 2 = x - 1$
$y = x + 1$

Example 2: The Ace Company installed a new machine in one of its factories at a cost of $20,000. The machine is depreciated linearly over 10 years with a scrap value of $2,000. Find the value of the machine after 5 years.

$m = \frac{\text{Scrap} - \text{Initial}}{\text{time}} = \frac{2{,}000 - 20{,}000}{10} = -1800$

$y = mx + b$

$V(t) = mt + \text{Initial}$
$= -1800t + 20{,}000$

$V(5) = -1800(5) + 20{,}000 = \boxed{\$11{,}000}$

Example 3: The AC Florist Company got a new delivery van at a cost of $28,000. The van is depreciated linearly over 5 years and has no scrap value. Find the value of the machine after 2 years.

$\$0$

$m = \frac{0 - 28{,}000}{5} = -5{,}600$

$V(t) = -5{,}600t + 28000$

$V(2) = -5{,}600(2) + 28{,}000 = \boxed{\$16{,}800}$

Example 4: 4. A manufacturer has a monthly fixed cost of $1200 and a production cost of $2.50 for each unit produced. The product sells for $10 per unit.

a. What is the cost function?

$C(x) = cx + F = 2.50x + 1200$

b. What is the revenue function?

$R(x) = sx = 10x$

c. What is the profit function?

$P(x) = R(x) - C(x) = (s - c)x - F = 7.5x - 1200$

d. What is the break- even point? (Break-Even Quantity, Break-Even Revenue)

$R(x) = C(x)$
$10x = 2.5x + 1200$
$7.5x = 1200$
$x = 160$

Break Even Quantity

$R(x) = 10x$
$R(160) = 10(160)$
$= 1600$ Break-Even Revenue

$\boxed{(160, 1600)}$

Example 5: Solve using Gauss-Jordon.

$$x - 5y + z = 5$$
$$-y + z = 2$$
$$3x + 2y + z = 11$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 3 & 2 & 1 & 11 \end{array}\right) \xrightarrow{-3R_1 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 0 & 17 & -2 & -4 \end{array}\right)$$

$-1 \cdot R_2 \to R_2$
$$\left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 17 & -2 & -4 \end{array}\right) \xrightarrow[-17R_2 + R_3 \to R_3]{5R_2 + R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 15 & 30 \end{array}\right) \xrightarrow{\frac{1}{15}R_3 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow[1 \cdot R_3 + R_2 \to R_2]{4R_3 + R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\boxed{\begin{array}{ll} x = 3 & \\ y = 0 & (3, 0, 2) \\ z = 2 & \end{array}}$$

Example 6: Given the following matrices are in row reduced form. State the solution, if it exists to the system of equations.

$$\begin{array}{ccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 6 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$x = -2$
$y = 1$
$z = 6$

Infinite
$x = 2$
$y + 2z = 0$ (Line)
$0 = 0$

$x = -1$
$y = 3$
$0 \neq 4$
No solution

Example 7: Solve for a, b, c and d.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix}$$

$a - 2 = 4$
$a = 6$

$b - (-1) = 3$
$b + 1 = 3$
$b = 2$

$c - (-5) = -2$
$c + 5 = -2$
$c = -7$

$d - 6 = 4$
$d = 10$

Example 8: Find the transpose of the following matrices.

Rows ⇒ Columns

$$A = \begin{bmatrix} -2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

Columns ⇒ Rows

$$B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2 & 1 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \qquad B^T = \begin{bmatrix} 2 & 0 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$$

Example 9: Find the product, if possible.

$$\begin{bmatrix}1 & -1 & 0 & 1\\ 2 & 1 & 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 0\\ -1 & 2\end{bmatrix}$$

$(2 \times 4)\ (3 \times 2)$

Don't Match

Not possible

Example 10: Find the product, if possible

$$\begin{bmatrix}2 & 1\\ 1 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -1\end{bmatrix}$$

$(3 \times 2)\ (2 \times 3)$

size of product = 3×3

$R_1 \cdot C_1$ $R_1 \cdot C_2$ $R_1 \cdot C_3$

$2+3$ $-2+0$ $4-1$

$R_2 \cdot C_1$ $R_2 \cdot C_2$ $R_2 \cdot C_3$

$1+0$ $-1+0$ $2+0$

$R_3 \cdot C_1$ $R_3 \cdot C_2$ $R_3 \cdot C_3$

$-1+6$ $1+0$ $-2-2$

$$\boxed{\begin{pmatrix}5 & -2 & 3\\ 1 & -1 & 2\\ 5 & 1 & -4\end{pmatrix}}$$

Example 11: Find the inverse of the following matrix

$$\begin{bmatrix}5 & 3\\ -4 & 6\end{bmatrix} \quad \begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$A^{-1} = \frac{1}{D}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

$D = ad - bc$

$= 5(6) - 3(-4) = 42$

$$A^{-1} = \frac{1}{42}\begin{bmatrix}6 & -3\\ 4 & 5\end{bmatrix} = \begin{bmatrix}\frac{6}{42} & \frac{-3}{42}\\ \frac{4}{42} & \frac{5}{42}\end{bmatrix} = \boxed{\begin{bmatrix}\frac{1}{7} & -\frac{1}{14}\\ \frac{2}{21} & \frac{5}{42}\end{bmatrix}}$$

Example 12: A manufacturer of stereo speakers, makes two kinds of speakers, an economy model which sells for \$50 and a deluxe model which sells for \$200. The deluxe model uses 1 woofer and 2 tweeters. The economy uses 1 woofer and 1 tweeter. The manufacturer currently has 20 woofers and 45 tweeters in inventory. Set-up the problem to maximize income from the sale, use x for economy and y for deluxe.

	x	y	
Woofers	1	1	≤ 20
tweeter	1	2	≤ 45
Profit	50	200	

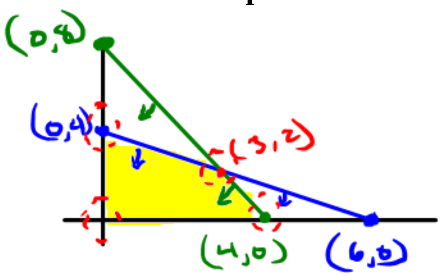
Max Profit $= 50x + 200y$

s.t. $x + y \le 20$

$x + 2y \le 45$

$x, y \ge 0$

Example 13: Maximize the following Linear Programming Problem.



$$\begin{aligned} Max\ P &= 3x + 2y \\ st\!:\ 2x + 3y &\le 12 \quad ① \\ 2x + y &\le 8 \quad ② \\ x, y &\ge 0 \end{aligned}$$

$2x+3y \le 12$
$3y \le -2x+12$
$y \le -\frac{2}{3}x + 4$

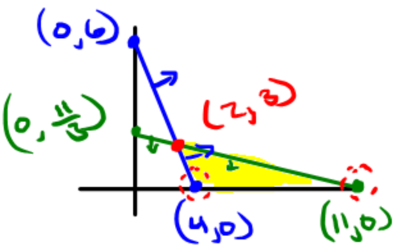
$2x+y \le 8$
$y \le -2x+8$

$-\frac{2}{3}x + 4 = -2x + 8$
$\frac{4}{3}x = 4$
$x = 3$
solve for y.
$y = 8 - 2x$
$y = 8 - 2(3)$
$= 2$

pts	$P = 3x + 2y$
(0,0)	$3(0) + 2(0) = 0$
(0,4)	$3(0) + 2(4) = 8$
(3,2)	$3(3) + 2(2) = 13$
(4,0)	$3(4) + 2(0) = 12$

Optimal value of 13 @ (3,2)

Example 14: Minimize the following Linear Programming Problem.



$$\begin{aligned} Min\ C &= x + y \\ st\!:\ 3x + 2y &\ge 12 \quad 1 \\ x + 3y &\le 11 \quad 2 \\ x, y &\ge 0 \end{aligned}$$

$3x+2y \ge 12$
$2y \ge -3x + 12$
$y \ge -\frac{3}{2}x + 6$

$x + 3y \le 11$
$3y \le -x + 11$
$y \le -\frac{1}{3}x + \frac{11}{3}$

$-\frac{3}{2}x + 6 = -\frac{1}{3}x + \frac{11}{3}$
$-\frac{9}{6}x + \frac{2}{6}x = \frac{11}{3} - \frac{18}{3}$
$-\frac{7}{6}x = -\frac{7}{3}$
$x = 2$
solve for y: $y = 3$

Pts	$x + y$
(2,3)	$2+3 = 5$
(11,0)	$11+0 = 11$
(4,0)	$4+0 = 4$

Optimal value is $C = 4$ @ (4,0)

Example 15: Find the accumulated amount at the end of 6 months on a \$2000 bank deposit paying simple interest at a rate of 3% per year.

$F = P(1 + rt)$

$P = 2000$
$r = 0.03$
$t = \frac{6}{12} = 0.5$

$F = 2000(1 + 0.03(0.5))$
$= \boxed{2030}$

$i = \frac{r}{m}$

$n = mt$

Example 16: Dave invested a sum of money 3 years ago in a savings account that has since paid interest at the rate of 4.5% per year compounded monthly. His investment is now worth $5,721.24. How much did he originally invest? ← P.V.

One time Deposit

P.V. w/ CI

$P = F(1+i)^{-n}$

$P = 5721.24(1+0.045/12)^{-36}$

$\boxed{= \$5000}$

Annuity

Example 17: Mike pays $300 per month for 4 years for a car, making no down payment. If the loan borrowed costs 7% per year compounded monthly, what was the original cost of the car? How much interest will be paid?

← P.V.

P.V.A

$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$

$P = 300(1-(1+0.07/12)^{-48})/(0.07/12)$

$= 12,528.06 + DP$

$300 \times 48 = 14,400$

$14,400 - 12,528.06 = \boxed{1871.94}$

Example 18: Steve bought a car for $30,000 . He put down 10% and financed the balance. His bank charged him 5% compounded monthly for 5 years. What is the monthly payment?

← E??

Amort. or Sinking

$E = \frac{Pi}{(1-(1+i)^{-n})}$

$P = 30000 - 10\%$

$30000 - 30000(0.1)$

$P = 27,000$

$E = \boxed{509.52}$

one time Deposit

Example 19: George decided to deposit $4,000 to pay for a cruise he plans to take in 2 years. His bank pays 3.5% annual interest compounded semiannually. How much will he have in his account at the end of two years? F.V.

$m=2$

FV w/ CI

$F = P(1+i)^n$ $= 4000(1+0.035/2)^4$ $= \boxed{\$4287.44}$

Example 20: Sandy decided to save some money for her daughter's college education. She decided to save $500 per quarter. Her credit union pays 4.5% annual interest compounded quarterly. How much money will she have available when her daughter starts college in 10 years? ← F.V. Annuity

quarter

$m = 4$

$F = E\left[\frac{(1+i)^n - 1}{i}\right]$

$\boxed{F = 25,093.42}$

Example 21: Let U={1,2,3,4,5,6,7,8,9,10},A = {1,3,5,7,9}, B={2,4,6,8,10}, C={1,2,4}

a. $B \cap C^c$

$\{2,4,6,8,10\} \cap \{3,5,6,7,8,9,10\}$

In Common $= \boxed{\{6,8,10\}}$

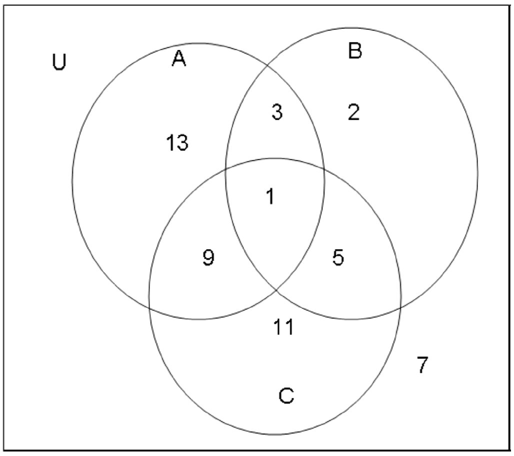
b. $A \cup B^c$

$\{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

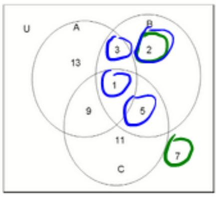
↑ All numbers listed

$\boxed{\{1,3,5,7,9\}}$

Example 22: Given the Venn Diagram.



a. $n[B \cup (A^C \cap C^c)]$ = $B \cup (A \cup C)^c$

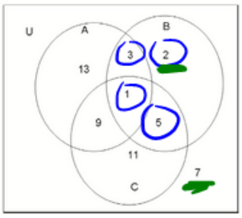


$3+2+1+5+7$

$= \boxed{18}$

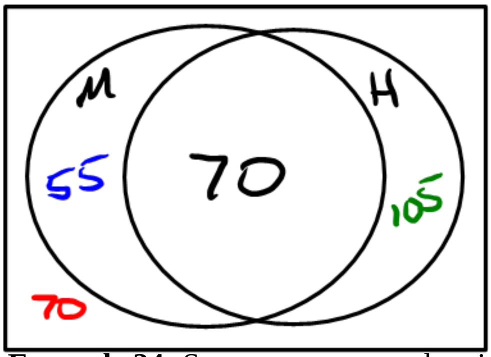
b. $n[B \cap (A^C \cap C^c)]$ $B \cap (A \cup C)^c$

In common



$= \boxed{2}$

Example 23: In a group of 300 hundred students, 125 are currently taking a math class and 175 are taking a history class and 70 are taking both classes. How many students in this group are taking a math class or a history class but not both?.



$55 + 105$

$= \boxed{160}$

Example 24: Suppose a person planning a banquet cannot decide how to seat 6 honored guests at the head table. In how many arrangements can they seated in the 6 chairs on one side of the table? Permutation P(# of chairs, # of guests)

$P(6,6) = \boxed{720}$

Example 25: In how many ways can a president, vice president, secretary, and treasurer be selected from an organization of 20 members?

Titles => Perm. $P(20,4) = \boxed{116{,}280}$

Example 26: You are going to make a serial number which can have no repeats and contains 3 digits and two letters. A zero cannot be the first digit. How many serials numbers are possible?

$\underset{1-9}{\underline{9}} * \underset{\text{Not the 1st num.}}{\underline{9}} * \underset{\text{Not the 1st 2}}{\underline{8}} * \underline{26} * \underline{25}$

$\boxed{421{,}200}$

Example 27: A car dealer is offering special pricing on a truck. It has four models, six exterior colors, 3 interior colors, four choices of seat coverings and 3 stereo systems. If you can only choose one in each category, how many different trucks could be constructed?

$$\underset{\text{models}}{\underline{4}} \quad \underset{\text{Ext.}}{\underline{6}} \quad \underset{\text{int}}{\underline{3}} \quad \underset{\text{seat}}{\underline{4}} \quad \underset{\text{stereo}}{\underline{3}} = \boxed{864}$$

Example 28: Find the number of ways in which 8 members of the space shuttle crew can be selected from 20 available astronauts. No order. Nothing significant

$$C(20,8) = \boxed{125,970}$$

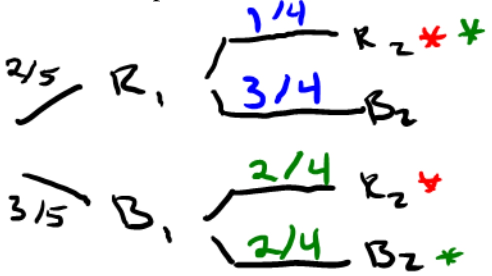
b. The command structure on a space flight is determined by the order in which astronauts are selected for the flight. How many different command structures are possible if 8 astronauts are selected from 20 that are available?

$$P(20,8) = \boxed{5,079,110,400}$$

c. If 14 men and 6 women are available for a space shuttle flight, in how many crews are possible that have 5 men and 3 women?

$$C(14,5) * C(6,3) = \boxed{40,040}$$

Example 29: A box contains 2 red marbles and 3 black marbles. Two marbles are drawn in succession without replacement. Find the following:



a. Find the probability the second marble is red?

$$P(\text{2nd is Red}) = \frac{2}{5}\cdot\frac{1}{4} + \frac{3}{5}\cdot\frac{2}{4} = \boxed{0.40}$$

b. Find the probability that both marbles are the same color?

$$P(\text{same color}) = \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) = \boxed{0.40}$$

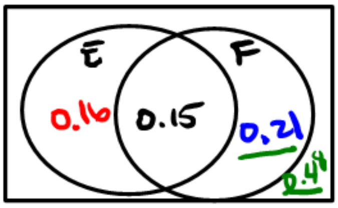
c. Find the probability that the second marble is black given the first marble is red?

$$P(B_2 \mid R_1) = \frac{P(B_2 \cap R_1)}{P(R_1)} = \frac{2/5\,(3/4)}{2/5} = 3/4 = \boxed{0.75}$$

d. Find the probability that first marble red given that the second marble is red?

$$P(R_1 \mid R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{2/5\,(1/4)}{\frac{2}{5}\left(\frac{1}{4}\right)+\frac{3}{5}\left(\frac{2}{4}\right)} = \boxed{0.25}$$

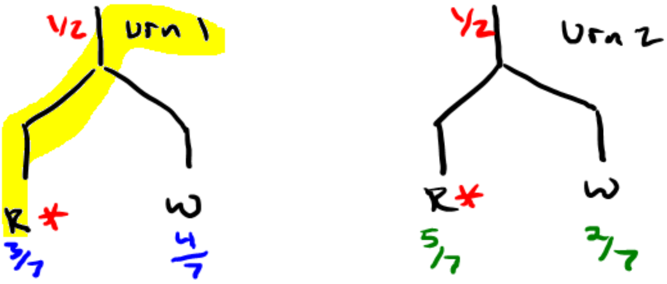
Example 30: Let E and F be events of a sample space S. Let $P(E^C) = 0.69$, $P(F) = 0.36$ and $P(E \cap F) = 0.15$. Find $P(E \cup F)$. ← Inside the circles



$0.15 + 0.21 + 0.44$
$= 0.84$
$1 - 0.44 = 0.16$

$0.16 + 0.15 + 0.21$
$= \boxed{0.52}$

Example 31: Urn I contains 3 red and 4 white marbles and Urn II contains 5 red and 2 white marbles. Each Urn has an equally likely probability of being chosen. Find the following probabilities if a marble is chosen:



a. What is the probability that Urn I is selected and a red marble?

$P(\text{Urn 1} \cap \text{Red}) = \frac{1}{2}\left(\frac{3}{7}\right) = \frac{3}{14} = \boxed{0.214}$

b. What is the probability that a red marble is chosen?

$P(\text{Red}) = 1/2\,(3/7) + 1/2\,(5/7) = \frac{4}{14} = \boxed{0.5714}$

c. What is the probability that Urn I is selected given that a red marble has been selected?

$P(\text{Urn 1} \mid \text{Red}) = \dfrac{P(\text{Urn 1} \cap \text{Red})}{P(\text{Red})} = \dfrac{1/2\,(3/7)}{1/2\,(3/7) + 1/2\,(5/7)} = \boxed{0.375}$

d. What is the probability that a white marble is chosen given that Urn II was selected?

$P(W \mid \text{Urn 2}) = \dfrac{P(W \cap \text{Urn 2})}{P(\text{Urn 2})} = \dfrac{1/2\,(2/7)}{1/2} = 0.2857$

Example 32: 30. A sample of 6 fuses is drawn from a lot containing 10 fuses and 2 defective fuses. Find the probability that the number of defective fuses is:

Total of 12

choosing 6 $\quad n(S) = C(12,6) = 924$

a. Exactly 1? $P(X=1) \Rightarrow$ 1 Def 5 Good $\quad \dfrac{C(2,1) * C(10,5)}{924} = \dfrac{504}{924} = \boxed{0.5455}$

b. No defective fuses?

2 Def $\quad$ 10 Good
0 $\quad$ 6

$\dfrac{C(2,0) * C(10,6)}{924} = \dfrac{210}{924} = \boxed{0.2273}$

c. At least 1 defective fuses?

← Use complement (0 Def)

$P(X \geq 1) = 1 - P(X=0)$

$1 - 0.2273 = \boxed{0.7727}$

Example 33: The probability distribution for a random variable X is given below. Calculate the expected value.

X	14	16	18	20
P(X=x)	0.34	0.31	0.26	0.09

$E(X) = x_1 p_1 + x_2 p_2 + \dots x_n p_n = 14(0.34) + 16(0.31) + \dots +$

$= \boxed{16.2}$

Example 34: Consider the following Binomial experiment. The probability that a new employee at a manufacturing plant is still employed after one year is 0.9. Seven people have recently been hired by the company. $p = 0.9 \quad q = 0.1 \quad n = 7$

$$P(X=x) = C(n,x)\, p^x q^{n-x}$$

a. What is the probability that exactly 4 of these new employees will still be employed after one year?

$P(X=4) = C(7,4)(0.9)^4(0.1)^3 = \boxed{0.02296}$

b. What is the probability that at least 6 of the new employee's will still be employed after one year? $P(X \geq 6) = P(X=6) + P(X=7)$

$= C(7,6)(0.9)^6(0.1)^1 + C(7,7)(0.9)^7(0.1)^0 = \boxed{0.85031}$

c. Calculate the mean of new employees that will still be employed after one year?

$E(X) = np = 7(0.9) = \boxed{6.3}$

d. Calculate the standard deviation.

$\sigma = \sqrt{var(X)} = \sqrt{npq} = \sqrt{7(0.9)(0.1)} = \boxed{0.7937}$

Example 35: Z is a standard normal random variable. Use the z-table

greater

a. Calculate P(Z > 0.19) . Use symmetry $= P(Z < -0.19) = \boxed{0.4247}$

b. Calculate P(–2.07 < Z < –1.63) . $P(Z < -1.63) - P(Z < -2.07)$

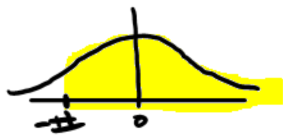
$0.0516 - 0.0192 = \boxed{0.0324}$

c. Find the z value, P(Z > z) = .9115 $\quad P(Z < -z) = 0.9115$ ← Look for it

greater than

$-z = 1.35 \quad \boxed{z = -1.35}$

d. Find the value of z, P(–z < Z < z) = .8444



$P(Z < z) = \frac{1}{2}\left(1 + P(-z < Z < z)\right)$

$= \frac{1}{2}(1 + 0.8444)$

$= \frac{1}{2}(1.8444)$

$= 0.9222$

↑ Look for this

$\boxed{z = 1.42}$

Example 36: Suppose X is a normal random variable with $\mu = 380$ and $\sigma = 20$. Find the value of:

a. $P(X < 405)$ $= P\left(Z < \frac{405-\mu}{\sigma}\right) = P\left(Z < \frac{405-380}{20}\right)$

$P(Z < 1.25) = \boxed{0.8944}$

b. $P(X > 330)$ $= P\left(Z > \frac{330-\mu}{\sigma}\right) = P\left(Z > \frac{330-380}{20}\right)$

$= P(Z > -2.50)$; Symmetry

$P(Z < 2.50) = \boxed{0.9938}$

Example 37: Use the normal distribution to approximate the following binomial distribution. Consider the random sample of 100 drivers on interstate 10 in Texas, where 29% of the drivers exceed the 70 mph speed limit. Find the probability that fewer than 40 drivers exceed the speed limit.

0.5 step

$n = 100$ $p = 0.29$ $q = 0.71$

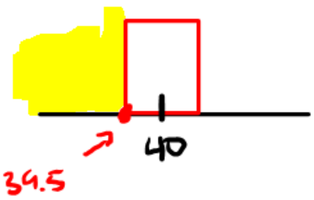
$P(X < 40) \approx P(Y < 39.5)$

$\mu = np$
$= 29$

$\sigma = \sqrt{npq}$
$= 4.5376$

$= P\left(Z < \frac{39.5-\mu}{\sigma}\right)$

$= P\left(Z < \frac{39.5-29}{4.5376}\right)$



$= P(Z < 2.3139)$

$P(Z < 2.31)$

$= \boxed{0.9896}$