Depreciation = decrease in the book value
Scap value = remaining value

Section 1.5A
Linear Functions and Math Models

Simple Depreciation

Example 1: In 2000, the B&C Company installed a new machine in one of its factories at a cost of \$250,000. The machine is depreciated linearly over 10 years with a scrap value of \$10,000.

slope = negative $y = mx + b$

a. Find the rate of depreciation for this machine.

time value
(0, 250000)
(10, 10000)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{scap value} - \text{initial value}}{\text{time}} = \frac{10000 - 250000}{10}$

$= \boxed{-24000}$

b. Find an expression for the machine's book value in the t-th year of use ($0 \le t \le 10$).

$V(t) = mt + \text{initial value}$ $V(t) = -24000t + 250000$

c. Find the machine's book value at the end of the 7th year.

$V(7) = -24000(7) + 250000 = \boxed{\$82000}$

Example 2: A company car has an original value of \$35,250 and it will be depreciated linearly over 5 years with a scrap value of \$7,000.

a. Find the rate of depreciation for this car.

slope

(0, 35250)
(5, 7000)

$m = \frac{7000 - 35250}{5} = -5650$

b. Find an expression for the car's book value in the t-th year of use ($0 \le t \le 5$).

$V(t) = -5650t + 35250$

Linear Cost, Revenue and Profit Functions

Let x be the number of units of a product manufactured or sold at a company then:

The **cost function**, $C(x)$, is the total cost of manufacturing x units of the product.

Fixed costs are costs that remain more or less constant regardless of the company's activity level.

Example: rental fees and executive salaries

Variable costs are costs that vary with production or sales.

Example: wages and costs for raw material

The **revenue function**, $R(x)$, is the total revenue realized from the sale of x units of the product.

The **profit function**, $P(x)$, is the total profit realized from manufacturing and selling x units of the product.

Formulas

Suppose a company has fixed cost of F dollars, production cost of c dollars per unit and selling price of s dollars per unit then

$C(x) = cx + F$
$R(x) = sx$
$P(x) = R(x) - C(x) = (s - c)x - F$

Given on Test 2!

where x is the number of units of the product produced and sold.

Example 2: A manufacturer has a monthly fixed cost of \$100,000 and a production cost of \$14 for each unit produced. The product sells for \$20 per unit.

a. Find the cost, revenue and profit functions.

$C(x) = 14x + 100000$

$R(x) = 20x$

$P(x) = R(x) - C(x) = 20x - (14x + 100000) = 6x - 100000$

b. Compute the profit (loss) corresponding to production levels of 15,000 units and 27,500 units.

$P(15000) = 6(15000) - 100000 = \$-10{,}000$ LOSS

$P(27500) = 6(27500) - 100000 = \$65000$ Profit

Example 3: A company that manufactures motorcycle helmets has monthly fixed costs of $55,000 and monthly cost of $21 per helmet. The selling price for each unit is $41.

a. How many helmets must the company produce and sell if they wish to make a profit of $50,000?

$$C(x) = 21x + 55000$$

$$R(x) = 41x$$

$$P(x) = 41x - (21x + 55000) = 20x - 55000$$

$$20x - 55000 = 50000$$

$$20x = 105000 \qquad x = \boxed{5250 \text{ helmets}}$$

b. What is the profit (loss) if they produce and sell 3500 helmets?

$$P(3500) = 20(3500) - 55000 = \$15000 \text{ Profit}$$