Section 2.2
Applications of Linear Programming Problems

Example 1: A company produces two models of clock radios. Each model A requires 15 min of work on assembly line I and 10 min of work on assembly line II. Each model B requires 10 min of work on assembly line I and 12 min of work on assembly line II. At most, 23 hr of assembly time on line I and 22 hr of assembly time on line II are available per work day. It is anticipated that the company will realize a profit of $12 on each model A and $10 on each model B. How many clock radios of each model should be produced per day in order to maximize the company's profit?

a. Define your variables.

x = # of model A   y = # of model B

b. Construct and fill-in the following table.

	x A	y B	MAX HR (at most)
ASMBLY LINE I	15 min	10 min	≤ 23 hours = 1380 min
ASMBLY LINE II	10 min	12 min	≤ 22 hours = 1320 min
Max **PROFIT**	$12	$10	

c. State the Linear Programming Problem. Do not solve.

Max $P(x,y) = 12x + 10y$

subject to
$$15x + 10y \le 1380$$
$$10x + 12y \le 1320$$
$$x \ge 0$$
$$y \ge 0$$

Example 2: A manufacturer makes camping tents, a standard model and a deluxe model. Each standard tent requires 1 labor-hour from the cutting department and 3 labor-hours from the assembly department. Each deluxe tent requires 2 labor-hours from the cutting department and 4 labor-hours from the assembly department. The maximum labor-hours available per week in the cutting department and the assembly department are 32 and 84, respectively. In addition, the distributor, because of demand, will not take more than 12 deluxe tents per week. If the company makes a profit of \$50 on each standard tent and \$80 on each deluxe tent, how many tents of each type should be manufactured each week to maximize the total weekly profit?

a. Define your variables.

x = # of standard tents y = # of deluxe tents

b. Construct and fill-in the following table.

	x **Standard**	y **Deluxe**	**Max hr**
Cutting Dept.	1 hour	2 hrs	≤ 32
Assembly Dept.	3 hrs	4 hrs	≤ 84
Max **Profit**	\$50	\$80	

c. State the Linear Programming Problem. Do not solve.

$$\text{Max } P(x,y) = 50x + 80y$$

$$\begin{aligned} \text{s.t.} \quad x + 2y &\le 32 \\ 3x + 4y &\le 84 \\ y &\le 12 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

Example 3: You're a dietician in a hospital and must arrange a special diet composed of two foods, Balanced Diet and Nutritional Goods. Each ounce of Balanced Diet contains 30 units of calcium, 10 units of iron, 10 units of vitamin A, and 8 units of cholesterol. Each ounce of Nutritional Goods contains 10 units of calcium, 10 units of iron, 30 units of vitamin A, and 4 units of cholesterol. If the minimum daily requirements are 360 units of calcium, 160 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to meet the minimum requirements and at the same time minimize the cholesterol intake? What is the minimum cholesterol intake?

a. Define your variables.

x = # of ounces of BD   y = # of ounces of NG

b. Construct and fill-in the following table.

	BD (x)	NG (y)	
Calcium	30	10	≥ 360
Iron	10	10	≥ 160
A	10	30	≥ 240
Min Cholesterol	8	4	

c. State the Linear Programming Problem. Do not solve.

$$\text{Min } Ch(x,y) = 8x + 4y$$

$$\text{s.t.} \quad \begin{aligned} 30x + 10y &\geq 360 \\ 10x + 10y &\geq 160 \\ 10x + 30y &\geq 240 \\ x, y &\geq 0 \end{aligned}$$

Example 4: The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and cost $100 to rent. The officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent in order to minimize the transportation costs? What are the minimal transportation costs?

a. Define your variables.

x = # of buses y = # of vans

b. Construct and fill-in a table.

	B (x)	V (y)	
Students	40	8	≥ 400 (at least)
Chaperons	3	1	≤ 36 (at most)
Min Cost	$1200	$100	

c. State the Linear Programming Problem.

$$\text{Min } C(x,y) = 1200x + 100y$$

$$\text{s.t. } \begin{aligned} 40x + 8y &\geq 400 \\ 3x + y &\leq 36 \\ x, y &\geq 0 \end{aligned}$$

$$40x + 8y = 400$$

$$5x + y = 50$$

d. Solve the problem.

Step 1: Graph the feasible set.

(1) $5x + y = 50$

0	50
10	0

$5x + y \geq 50$

$(0,0)$

$5(0) + 0 \not\geq 50$

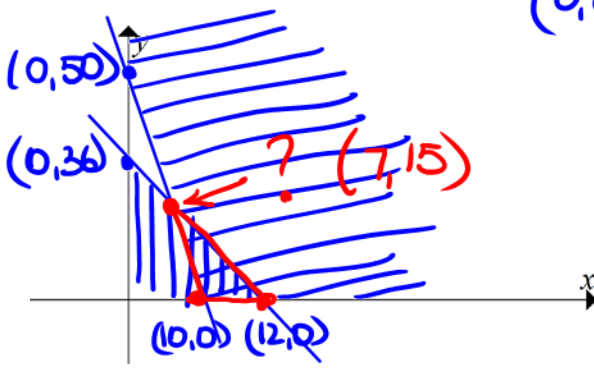
(2) $3x + y = 36$

0	36
12	0

$3x + y \leq 36$

$(0,0)$ ✓

$3(0) + 0 \leq 36$



Step 2: Find the corner points of the feasible set. $(10,0)$ $(12,0)$ $(7,15)$

$$\begin{aligned} + \quad 5x + y &= 50 \\ -3x - y &= -36 \\ \hline 2x &= 14 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 5(7) + y &= 50 \\ 35 + y &= 50 \\ y &= 15 \end{aligned}$$

Step 3: Find where the optimal solution occurs and the optimal value. Interpret your results.

Corner Points	Min $1200x + 100y$
$(10,0)$	$1200(10) + 100(0) = 12000$
$(7,15)$	$1200(7) + 100(15) = \boxed{9900 \Leftarrow \text{min}}$
$(12,0)$	$1200(12) + 100(0) = 144000$

7 buses + 15 vans

Example 5: A patient in a hospital is required to have at least 84 units of drug D_1 and at least 120 units of drug D_2 each day (assume that an overdosage of either drug is harmless). Two substances, M and N, contain each of these drugs; however, in addition, both contain an undesirable drug D_3. Each gram of substance M contains 10 units of drug D_1, 8 units of drug D_2 and 3 units of drug D_3. Each gram of substance N contains 2 units of drug D_1, 4 units of drug D_2 and 1 unit of drug D_3. How many grams of substances M and N should be mixed to meet the minimum daily requirements and at the same time minimize the intake of drug D_3?

a. Define your variables.

x = # of grams of M   y = # of grams of N

b. Construct and fill-in the following table.

	M (x)	N (y)	
D_1	10	2	$\geq$ 84
D_2	8	4	$\geq$ 120
Min D_3	3	1	

c. State the Linear Programming Problem.

$$\text{Min } D_3 = 3x + y$$

$$\text{s.t. } 10x + 2y \geq 84 \rightarrow 5x + y \geq 42$$

$$8x + 4y \geq 120 \rightarrow 2x + y \geq 30$$

$$x \geq 0$$

$$y \geq 0$$

d. Solve the problem.
Step 1: Graph the feasible set.

(1) $5x + y = 42$

$5x = 42$

0	42
8.4	0

$5x + y \geq 42$

$(0,0)$

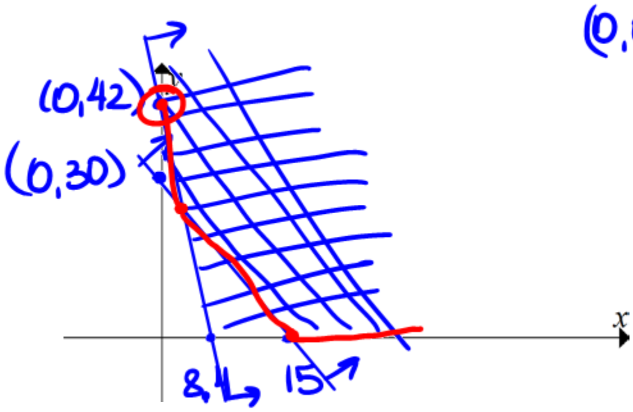
$5(0) + 0 \ngeq 42$

(2) $2x + y = 30$

0	30
15	0

$2x + y \geq 30$

$(0,0)$ $2(0) + 0 \ngeq 30$



Step 2: Find the corner points of the feasible set. $(0,42)$ $(15,0)$ $(4,22)$

$$\begin{array}{rl} & 5x + y = 42 \\ + & -2x - y = -30 \\ \hline & 3x = 12 \\ & x = 4 \end{array}$$

$5(4) + y = 42$

$20 + y = 42$

$y = 22$

Step 3: Find where the optimal solution occurs and the optimal value. Interpret your results.

$D_3 = 3x + y$

Corner Points	Min
$(0, 42)$	$3(0) + 42 = 42$
$(4, 22)$	$3(4) + 22 = \boxed{34 = \text{min}}$
$(15, 0)$	$3(15) + 0 = 45$

4 grams of M & 22 grams of N