Section 4.2
Annuities

A sequence of equal periodic payments made at the end of each payment period is called an **ordinary annuity.**

Examples of annuities:

1. Regular deposits into a savings account.
2. Monthly home mortgage payments.
3. Payments into a retirement account.

We will study annuities that are subject to the following conditions:

1. The terms are given by fixed time intervals.
2. The periodic payments are equal in size.
3. The payments are made at the end of the payment periods.
4. The payment periods coincide with the interest conversion periods.

The sum of all payment made and interest earned on an account is called the **future value of an annuity.**

Future Value of an Annuity

The future value F of an annuity of n payments of E dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$\boxed{F = E\left[\frac{(1+i)^n - 1}{i}\right]}$$

$i = \frac{r}{m}$

$n = mt$

Present Value of an Annuity

The present value P of an annuity of n payments of E dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$\boxed{P = E\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

Example 1: Carrie opened an IRA on January 31, 1990, with a contribution of $2000. She plans to make a contribution of $2000 thereafter on January 31 of each year until her retirement in the year 2009 (20 payments). If the account earns interest at the rate of 8% per year compounded yearly, how much will Carrie have in her account when she retires?

PV Annuity

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

FV Annuity

$$\boxed{F = E\left[\frac{(1+i)^{n}-1}{i}\right]}$$

$$F = 2000\left[\frac{\left(1+\frac{0.08}{1}\right)^{20}-1}{.08/1}\right] = \$91,523.93$$

2000 (((1 + .08) ^20 - 1) / .08)

Example 2: Alfreda pays $320 per month for 4 years for a car, making no down payment. If the loan borrowed costs 6% per year compounded monthly, what was the cash price of the car?

PV Annuity

$$\boxed{P = E\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

FV Annuity

$$F = E\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$P = 320\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-48}}{0.06/12}\right]$$

$$= \$13,625.70$$

$$320(48) = 15360$$

320 (((1 - (1 + .06 / 12) ^ (- 48)) / (.06 / 12)))

Example 3: Donald and Daisy paid $10,000 down toward a new house. They decided to finance the rest and so they have a 30-year mortgage for which they pay $1,100 per month. If interest is 6.35% per year compounded monthly, what was the purchase price of the house?

$$\boxed{\text{PV Annuity} \quad P = E\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

FV Annuity

$$F = E\left[\frac{(1+i)^{n}-1}{i}\right]$$

$i = \frac{0.0635}{12}$ $\quad n = 30(12) = 360$

$$P = 1100\left[\frac{1-\left(1+\frac{0.0635}{12}\right)^{-360}}{(0.0635/12)}\right] = 176,781.88$$

$$176,781.88 + 10000 = \boxed{\$186,781.88}$$

+ downpayment

Example 4: Gary decided to save some money for his daughter's college education. He decided to save $300 per quarter. His credit union pays 4.5% per year compounded quarterly. How much money will he have available when his daughter starts college in 10 years?

PV Annuity

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\boxed{\text{FV Annuity} \quad F = E\left[\frac{(1+i)^{n}-1}{i}\right]}$$

$$F = 300\left[\frac{\left(1+\frac{0.045}{4}\right)^{40}-1}{(0.045/4)}\right] \quad \$15,050.05$$

Example 5: You buy an entertainment system from Ernie's Electronics on credit. If your monthly payments are $135.82 and the store charges 15% per year compounded monthly for 2 years, what was the original cost of the entertainment system?

PV Annuity

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

FV Annuity

$$F = E\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$P = 135.82\left[\frac{1-\left(1+\frac{0.15}{12}\right)^{-24}}{(0.15/12)}\right] = \$2\,801.18$$

Example 6: Barry wishes to set up an account for his grandfather so that he can have some extra money each month. Barry wants his grandfather to be able to withdraw $120 per month for the next 4 years. How much must Barry invest today at 4% per year compounded monthly so that his grandfather can withdraw $120 per month for the next 4 years?

PV Annuity

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$n = mt = 12(4) = 48$$

FV Annuity

$$F = E\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$P = 120\left[\frac{1-\left(1+\frac{0.04}{12}\right)^{-48}}{(0.04/12)}\right] = \$5{,}314.66$$