

Section 4.3 Amortization and Sinking Funds

To **Amortize** means to pay off a debt by installment payments.

Amortization Formula

The periodic payment E on a loan of P dollars to be amortized over n periods with interest charged at the rate of i per period is

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

A fund that is set up for a specific purpose at some future date is called a **sinking fund**.

Sinking Fund Formula

The periodic payment E required to accumulate a sum of F dollars over n periods with interest charged at the rate of i per period is

$$E = \frac{iF}{(1 + i)^n - 1}$$

Example 1: Kelly wishes to buy a car that costs \$32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 60 months. She decides to secure the loan from the dealer. How much will her monthly payments be?

Amortization

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1 + i)^n - 1}$$

$$E = \frac{32998 \left(\frac{0.0625}{12} \right)}{1 - (1 + 0.0625/12)^{-60}} = \$641.79$$

Example 2: A person would like to have \$200,000 in an account for retirement 15 years from now. How much should be deposited **quarterly** in an account paying 6% per year compounded quarterly to obtain this amount?

Amortization

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1 + i)^n - 1}$$

$$E = \frac{\frac{0.06}{4} (200000)}{\left(1 + \frac{0.06}{4}\right)^{60} - 1} = \$2078.69$$

Example 3: The cash price of the sailboat you wish to purchase is \$16,000. You pay **15% down** and secure a loan for the remaining balance. How much are your monthly payments if 18% per year compounded **monthly** is charged over a period of 6 years?

Amortization

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1 + i)^n - 1}$$

$$16000(.85) = 13600$$

$$E = \frac{13600 \left(\frac{.18}{12}\right)}{1 - \left(1 + \frac{.18}{12}\right)^{-72}} = \$310.19$$

Example 4: A company anticipates needing to replace one of their assembly line machines in 3 years at a cost of \$25,000. An account at their credit union pays 4.5% per year compounded monthly. How much would they need to deposit in this account each month to reach their goal in 3 years?

Amortization

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1 + i)^n - 1}$$

$$E = \frac{\left(\frac{0.045}{12}\right) 25000}{\left(1 + \frac{0.045}{12}\right)^{36} - 1} = \$649.92$$

Example 5: In order to expand their growing business, a company purchased a piece of property at a cash price of \$450,000. They made a \$25,000 down payment, and financed the rest through a lender. If the lender will charge 3.25% per year compounded semiannually for 7 years, find the company's semiannual payment.

Amortization

$$E = \frac{Pi}{1 - (1 + i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1 + i)^n - 1}$$

$$P = 450000 - 25000 = 425000$$

$$E = \frac{425000 \left(\frac{0.0325}{2}\right)}{1 - \left(1 + \frac{0.0325}{2}\right)^{-14}} = \$34186.03$$

Try this one: You bought an RV at a cost of \$125,000. You made a down payment of \$3,000. How much are your monthly payments to the creditor over 20 years at 2.25% per year compounded monthly?