Section 6.4
Use of Counting Techniques in Probability

Let S be a uniform sample space and let E be any event. Then, $P(E) = \frac{n(E)}{n(S)}$.

Example 1: Consider the experiment of tossing a fair coin 10 times.

a. Find the probability that the coin lands heads exactly 7 times.

$n(S) = 2^{10} = 1024$

$n(E) = C(10,7) = 120$

$P(7H) = \frac{120}{1024} \approx .12$

b. Find the probability that the coin lands heads at most 2 times.

0H or 1H or 2H

$n(E) = C(10,0) + C(10,1) + C(10,2)$

$= 1 + 10 + 45 = 56$

$P(\text{at most 2H}) = 56/1024 \approx .05$

c. Find the probability that the coin lands tails at least 9 times.

9T or 10T

$n(E) = C(10,9) + C(10,10) = 10 + 1 = 11$

$P(\text{at least 9T}) = \frac{11}{1024} \approx 0.01$

d. Find the probability that the coin lands tails at least once.

1 or 2 or 3 . . . or 10

complement

OT

No tails $= C(10,0) = 1$

$\frac{1024 - 1}{1024} = \frac{1023}{1024} \approx .999$ ← # of outcomes with at least 1T

$1 - \frac{1}{1024} = \frac{1024}{1024} - \frac{1}{1024} = \frac{1023}{1024} \approx .999$

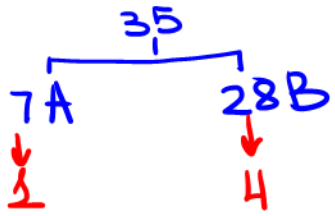
Example 2: In a survey of 35 consumers at a local supermarket, 7 indicated that they buy brand A of a certain product and the rest indicated that they buy brand B of the same product. You choose 5 customers surveyed at random.

a. What is the probability that 4 buy brand B and 1 buys brand A?

$n(E) = C(7,1)\,C(28,4)$

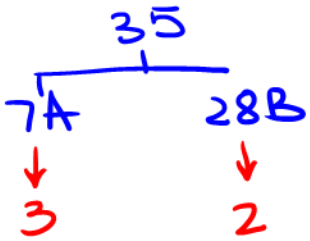
$n(S) = C(35,5)$

$P(E) = \dfrac{C(7,1)\,C(28,4)}{C(35,5)} \approx .44$



b. What is the probability that 3 buy brand A?

$P(E) = \dfrac{C(7,3)\,C(28,2)}{C(35,5)} \approx 0.04$



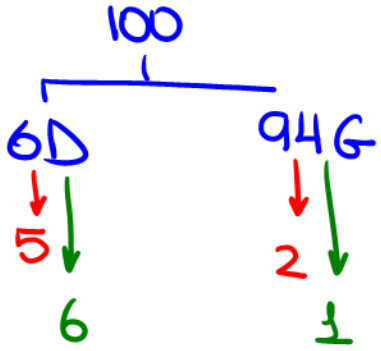
Example 3: A department store is shipped 100 remote controlled cars of which 6 are defective. A customer selects 7 cars at random.

a. What is the probability that at least 5 will be defective?

5 or 6

$P(E) = \dfrac{C(6,5)\,C(94,2) + C(6,6)\,C(94,1)}{C(100,7)}$

$\approx .0000016$



b. What is the probability that at most 4 will be defective?

0, 1, 2, 3, 4 complement 5 or 6

$\dfrac{C(100,7) - [C(6,5)\,C(94,2) + C(6,6)\,C(94,1)]}{C(100,7)} \approx .999998$

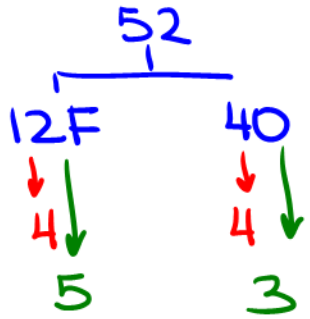
OR

$1 - \dfrac{C(6,5)\,C(94,2) + C(6,6)\,C(94,1)}{C(100,7)}$

Example 4: Eight cards are selected at random from a well-shuffled deck of 52 playing cards.

a. What is the probability that either 4 face cards or 5 face cards are chosen?

$$\frac{C(12,4)C(40,4)+C(12,5)C(40,3)}{C(52,8)} \approx .18$$



c. What is the probability that 5 cards are red?

$$\frac{C(26,5)C(26,3)}{C(52,8)} \approx .23$$