Section 7.4
The Binomial Distribution

A binomial experiment has the following properties:

1. The number of trials is fixed.
2. There are two outcomes of the experiment: Success, with probability p and Failure, with probability q. Note: $p + q = 1$.
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment
In a binomial experiment in which the probability of success in any trial is p, the probability of exactly x successes in n independent trials is given by

$$P(X = x) = C(n, x)\, p^x q^{n-x}$$

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**.

$n = 10$

Example 1: An experiment consists of 10 independent trials where the probability of success is $\frac{5}{8}$. Find each of the following probabilities. $p = \frac{5}{8}$ $\quad q = 1 - p = 1 - \frac{5}{8} = \frac{3}{8}$

a. The probability of obtaining exactly $\overset{x}{5}$ successes.

$$P(X=5) = C(10,5)\left(\frac{5}{8}\right)^5\left(\frac{3}{8}\right)^5 = .1782$$

b. The probability of obtaining at least 1 success. 1 or 2 or ... 10 complement 0 Successes

$$P(\text{at least } 1\,S) = 1 - P(0\,S)$$
$$= 1 - \cancelto{1}{C(10,0)}\cancelto{1}{\left(\frac{5}{8}\right)^0}\left(\frac{3}{8}\right)^{10} = 1 - \left(\frac{3}{8}\right)^{10}$$
$$= .9999$$

c. $P(X \le 1)$

$$= P(X=0) + P(X=1)$$
$$= C(\overset{n}{10},\overset{x}{0})\left(\frac{5}{\underset{p}{8}}\right)^{\overset{x}{0}}\left(\frac{3}{\underset{q}{8}}\right)^{\overset{n-x}{10}} + C(\overset{n}{10},\overset{x}{1})\left(\frac{5}{\underset{p}{8}}\right)^{\overset{x}{1}}\left(\frac{3}{\underset{q}{8}}\right)^{\overset{n-x}{9}}$$

$= .00097$

Mean, Variance and Standard Deviation of a Random Variable

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p, and probability of failure q, then the mean E(X), variance and standard deviation of X are given by applying the following formulas:

$$\boxed{\begin{aligned} \mu &= np \\ Var(X) &= npq \\ \sigma &= \sqrt{Var(X)} \end{aligned}}$$

Example 2: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.84, what is the probability that out of 6 couples just married, in the next 20 years:

S = Divorse
F = No divorse
$n = 6$
$p = .84$
$q = .16$
$q = 1 - p$

a. none will be divorced?

$P(X=0) = C(6,0)(.84)^0(.16)^6 = 0.000017$

b. all will be divorced?

$P(X=6) = C(6,6)(.84)^6(.16)^0 = .3513$

c. Find the mean and standard deviation of the experiment.

$\mu = np = 6(.84) = 5.04$

$\sigma = \sqrt{npq} = \sqrt{6(.84)(.16)} = .898$

Example 3: Consider the following binomial experiment. It is estimated that 34% of the general population has blood type A^+. If a sample of 9 people is selected at random, what is the probability that at least 8 (8 or 9) of them have blood type A^+?

$n = 9$
$p = .34$
$q = .66$

$P(X=8) + P(X=9)$

$= C(9,8)(.34)^8(.66)^1 + C(9,9)(.34)^9(.66)^0$

$= 0.0011$

Example 4: The probability of a person contracting influenza on exposure is 62%. In the binomial experiment for a group of 12 people that has been exposed, what is the the probability that at most 10 (0, 1, 2, ..., 10) contract influenza?

complement 11 or 12

$n = 12$
$p = .62$
$q = .38$

$1 - [P(X=11) + P(X=12)]$

$= 1 - C(12,11)(.62)^{11}(.38)^1 - C(12,12)(.62)^{12}(.38)^0$

$= .973$